CHAPTER FOUR

# Towards the Logic of General Names

## 1. From syllogistic to the calculus of classes

**1.1.** The preceding chapter tells of programmes of reforming logic, all of them being concerned with what we now call **traditional logic**. By the end of the 19th century there appears a new kind of logic. It was so new that people needed to call it by a new name.

The term *logistic* dated from the International Congress of Philosophy of 1904, where it was suggested independently by Itelson, Lalande, and Couturat. Louis Couturat, the renowned discoverer of Leibniz's logic, might have followed Leibniz who employed either the name *calculus ratiocinator*, or *logica mathematica*, or else *logistica* for his logical systems.

That expression, after a fairly wide reception in the thirties, later went out of use. The designation **mathematical logic**, also following Leibniz, proved more lasting but nowadays it denotes only a part of what was originally meant, viz., the logical study of mathematical systems, being distinguished from the logical study of philosophical systems; at present, the latter happens to be called **philosophical logic**.

Many authors and institutions prefer the term *symbolic logic* as sufficiently general and rendering the concern with the symbolic aspect of proof procedures, while in some historical contexts, it is the expression **modern logic** which proves more convenient.[1] As far as the difference between the old and the modern logic is concerned, the opinions of historians and philosophers of logic may vary from stressing their radical incompatibility to the claim that traditional logic forms just a tiny part of the modern version.

[1] See, e.g., *Logic* [1981], "Logic, modern, history of" by W. Marciszewski.

Apart from philosophical motivations, such differences may depend on views concerning the nature of historical development. Those who see the development of ideas as a dramatic process, involving sudden turns and revolutions, are more inclined to stress the novelty of modern logic, while more conservative minds perceive continuity.

It is the contention of this Chapter that syllogistic paved the way for the calculus of classes and that this, in turn, contributed to what can be called the **computational formalization** of logic. This stage of formalization has become possible due to the algebraization of logic claimed since the 17th century (as extensively reported in Chapter Three) which was a continuous process; it reached its mature stage in Boolean algebra in the middle of the 19th century. This algebra has become the paradigm of modern computational logic (*Boole's rechnende Logik*, as Frege called it); its theories duly deserve the name of calculi. As for the feature of formal restructuring, that is a new conception of logical form, the change was more rapid and unexpected, due to Frege's modelling of the structure of logical formulas on the mathematical language instead of imitating the syntax of natural languages.

Both points mentioned above are of consequence for the main problem of this essay, that is the problem of what modern logic can contribute to cognitive rhetoric (cf. Chapter One, Subsec. 1.3). It is worth noting that whenever rhetoric flourished in the past, it had close links with traditional logic. Is it possible to create comparably strong links between rhetoric and modern logic? To work out premises for answering this question, in the subsections following this one I shall briefly discuss the issues of computational formalization and formal restructuring.

**1.2.** The program of computational formalization of logic was much in vogue in the 17th century (as reported in the preceding chapter). It started with Thomas Hobbes who compared reasoning to computing, and culminated with Leibniz, who in his numerous calculi tried to reduce reasoning to counting (his famous *calculemus* as a recipe for problem-solving). In order to yield the idea of computational formalization, this trend towards computing in logic must have met with another one, that going back to the

Schoolmen, whom Leibniz appreciated for their notion of logical form (cf. Three, Subsec. 3.1).

That amounted to developing logic as a theory of particular physical objects, such as shapes of expressions, contrary to the Cartesian conception of logic as dealing with the behaviour of the mind. This should be construed as the processing of some physical objects as representatives of abstract logical objects as are, for example, truths. It is the very core of **formalization** that results which hold in an abstract domain are produced in an indirect way, namely that consisting in processing suitably coordinated physical entities into abstract ones.

These two features, computization and formalization, can be combined in a natural way, yet they are independent of each other. One can develop mathematics in a non-formalistic way, as do contemporary intuitionists, and one can adopt formalization procedures which are not computational, e.g., formalized religious rites. That stream in the history of logic which united these features has been finally crowned with the mechanization of reasoning — desired, planned, and envisaged by Leibniz (cf. Chapter Three, Subsec. 3.3). **Mechanization** is a special kind of **formalization**, namely such that linguistic forms are no longer figures produced with something like a pencil, but are configurations of physical impulses which belong to the functioning of a machine, and at the same time can be interpreted by humans; for instance, an electric pulse is interpreted as number one, its lack as number 0, while their sequences produce all numbers rendered in binary arithmetical notation. Owing to such correspondence between numbers as abstract entities and electric pulses as physical entities, machines can be commissioned to compute and to reason. The latter proved possible owing to an ingenious reduction of reasoning to computing.

Such a reduction was first attempted and accomplished within the field of traditional logic at some advanced stage of its development. The feasibility of such transformation depends on the way of interpreting the four forms of **general statement** recognized by Aristotle, and regarded by him and his followers as the complete classification of syntactic forms to be employed in logic. They are general in the sense that they consist of general names, namely two

names combined by the copula. Thus no sentence concerning individuals, that is containing an individual name as its grammatical subject, belongs to that syntactic pattern of traditional logic. In this logic, no logical rules deal with inferences which involve names referring to individuals as individuals (there were only attempts to refer them to one-element classes). This feature should catch due attention when compared with the fact that modern logic takes just the opposite approach; namely, the basic syntactic pattern is that of a sentence consisting of an individual name and a predicate, i.e., an **a**tomic sentence. This sheds light on the scale of the formal (i.e., syntactic) restructuring having been brought by the transition to modern logic.

However, the failure in completing the repertoire of logically relevant syntactic forms proved advantageous at a certain stage of development; it gave logic that simplicity which encouraged a computational approach with those tools alone which were at hand in that phase. Such an approach became possible after the mediaeval logicians developed the theory of *distributio terminorum* which provided a device to check the validity of syllogistic reasonings. It was the beginning of the extensional interpretation of general sentences, i.e., that in which both subject and predicate are taken as names of classes.[2]

Thus in the 17th century, when both scholastic logic and algebraic methods were perfectly assimilated by many philosophers,

[2] The theory of term distribution, to a great extent due to William of Shyreswood (d. 1249), defined the ways in which a general term may be taken, i.e., whether in its full extension or partial extension. Thus it belonged to the theory of *suppositio*, i.e., the examining of ways in which a term may be taken (as an individual, a class, etc.), but in another respect it contributed to the extensional conception of logic, admired by Leibniz for its technical advantages. Within this framework even the delicate problem of individual terms was solved, namely in the way suggested by William of Ockham (ca. 1300–1350) who treated an individual term as taken in its full extension, hence behaving as a general term; this paved the way for the set-theoretical concept of unit class (cf. Bocheński [1956], item 34.02). Aristotle himself rather ignored the problem of semantic interpretation of general statements, having had enough reasons to be satisfied with his technical achievement. As to possible semantical interpretations of Aristotle's general statements, see Kneale and Kneale [1962], II, 5; as to theories of distribution and supposition, see *ibid., passim.*

the time was ripe to attempt at an algebraic computization of logic. This was first done by Leibniz, but his results did not influence later authors since they remained unknown up to the end of the 19th century; only then were Leibniz's logical manuscripts discovered by Louis Couturat.[3]

More luck was had by Johann Heinrich Lambert (1728–1777), who admired Leibniz's logical genius but did not know anything about his algebraization of logic and worked it out by himself to publish it in his many logical volumes (at least the seven ones which are available at present), especially one under the much telling title *Neues Organon oder Gedanken über die Erforschung und Bezeichnung des Wahren*, published in Leipzig 1764 (cf. Lambert [1782], the chapter *Versuch einer Zeichenkunst in der Vernunftlehre*). Such terms as *Bezeichnung* und *Zeichenkunst* hint at the tendency towards formalization, while the phrase *Novum Organon* indicates the historical role of the work, seen as a counterpart to the *Organon* of Aristotle. In the same century there were more authors who tried an algebraic computization of logic, but it is a long story which should be told at another place.[4] Lambert deserves to be mentioned as a convincing evidence that similar results may be obtained in the same historical period independently of each other, as if they were governed by an objective law of development holding in a realm of abstract ideas.

**1.3.** In the 19th century the algebraical calculus of logical objects reached its maturity, again with many authors acting simultaneously. The most lucky among them proved to be the British mathematician George Boole [1847], [1854]. He created an advanced and viable theory which entered the history of logic under his name. Boolean algebra has become the first complete paradigm of computational logic and an indispensable tool in other branches of science and technology including computer science and the study of brain activity.

[3] An instructive exposition of Leibniz's results is found in Kneale and Kneale [1962], and in the editor's comments to the critical edition, accompanied by a German translation of Leibniz's *Generales inquisitiones de analysi notionum et veritatum*, Schupp [1982].

[4] Many important data on this subject can be found in Risse [1964] and in Styazhkin [1969].

In Boole's algebra, likewise in the earlier attempts of giving logic a calculative form, the Aristotelian syllogistic dealing with extensionally interpreted general statements proved a suitable laboratory for the computational treatment of logic. First, however, one needs to have cleverly invented those values which may appear in equations as results of operations performed on classes. Such values can be clearly defined due to the concept of the **universe of discourse**. For a while let us focus our attention upon that concept as introduced by Augustus de Morgan (1806–1871), another English pioneer of algebraical logic.[5]

A universe of discourse is what contains all the entities to be discussed in a given discourse, investigation or theory. These entities can be grouped into classes included in (i.e., being subclasses of) the universe in question. Now let us distinguish two subclasses, the greatest and the least, from among those included in the universe. Obviously, the greatest equals the universe itself while the least is one having zero elements; let them be called the **universal class** and the **empty class**, respectively. In principle, it is of no consequence what symbols we choose to denote these limiting cases, but in practice it proved convenient to use the symbol '0' for the empty class and '1' for the universal class. For some partial analogies with arithmetical operations help to interpret logico-algebraical equations in which these values appear as results of operations upon classes.[6]

To render a general statement in algebraic form we still need the equation sign and signs for two operations called complementation and intersection, both perfectly compatible with our handling of classes in everyday language.[7]

[5] De Morgan's calculus in his *Formal Logic* of 1847 is essentially similar to that of Boole but not so fully worked out, esp. in the notation and applications.

[6] Leibniz vaguely anticipated these two values. In his terminology the word *ens* would have corresponded to 1, and the word *non-ens* to 0.

[7] This is worth noting from the rhetorical point of view as confronting logical calculi with everyday arguments. In this context 'everyday language' should mean at least the Indoeuropean languages. The outstanding Polish logician Roman Suszko (d. 1979) used to repeat that mathematics, as based on the idea of *class*, could only have developed so enormously in the environment of those ethnic languages which involve the notion of class at the base of their syntactic structures, as do Greek, Latin, etc.

The **complementation** of a class, say $A$, is relative to the universe in question, say $U$; namely the complementation of $A$, symbolically, $-A$, is the class of all those and only those elements of $U$ which are not elements of $A$, that is all the rest which remains after 'removing' $A$ from $U$.

The *intersection* of classes, say $A$ and $B$, symbolically $A \cdot B$, is the class of all those and only those entities which are elements of both $A$ and $B$.

The Aristotelian general statements take the following forms.

1. *universal affirmative*:
Every $A$ is $B$, e.g., every masculinist is a male.
2. *universal negative*:
No $A$ is $B$, e.g., no masculinist is a male.
3. *particular affirmative*:
Some $A$ is $B$, e.g., some masculinists are male.
4. *particular negative*:
Some $A$ is not $B$, e.g., some masculinists are not male.[8]

Here are the corresponding Boolean equations:

(1) $A \cdot -B = 0$
(2) $A \cdot B = 0$
(3) $A \cdot B \neq 0$
(4) $A \cdot -B \neq 0$.

Now we can check the validity of some logical theorems in a purely computational way. For instance, there are theorems that (1) is equivalent to the denial of (4), and (2) is equivalent to the denial of (3); that two statements are equivalent means that they have the same truth-value, i.e., either both are true or both are false. Obviously, to deny (1) means to replace the equality sign by the sign of inequality, and then one obtains (4). The same transformation holds for (2) and (3). Two statements such that

[8] I am not sure whether the term 'masculinist' exists in English. This uncertainty is deliberately left as this gives opportunity to show that the understanding of terms substituted for schematic letters is irrelevant for understanding the logical relations between considered sentences. However, those who feel better when understanding all the constituents of the statement in question may fancy the definition that a masculinist is a member of the Men's Liberation Movement.

one of them is equivalent to the denial of the other are said to be **contradictory**.

Using transformations of this and similar kind one can demonstrate the validity of Aristotelian syllogisms, e.g. that famous one (as opening the list of syllogistic forms in all expositions): *every masculinist is a male; every male is fearless; hence, every masculinist is fearless.* The application of the class calculus to proving the validity of such a form of syllogistic argument requires a more systematic introduction which would lead us away from the course of the present discussion.[9]

In order to perform some other calculations we need the principle that the empty class has no members in common with any other class; in other words, *the intersection of the empty class with any class $A$ equals the empty class*, in symbols: $0 \cdot A = 0$. One can hardly find anything more obvious than this principle: the empty class as having no members at all cannot share any members with another class. This obviousness is worth emphasis from the rhetorical point of view, it should makes us aware of how close the calculus of classes is to our everyday thinking.

It is equally obvious that any statement of the form 1 tells us that there are no $A$s not being $B$s (i.e., their intersection is empty), e.g., *every cuckoo is a bird* means that there is no cuckoo being a non-bird, i.e., the class of cuckoos not being birds is empty.

However, these obvious assertions have a consequence which may seem shocking to everyday language users. For instance, those who do not believe in witches would be compelled to agree that, say, *every witch is an accomplice of the devil.* Indeed, if the class of witches is empty, then its intersection with any class whatever is empty: if there are no witches at all, then there are no witches plotting with the devil. Thus the statement in question proves true (under the weak interpretation of a universal statement).

## 2. The existential import of general names

**2.1.** The conclusion of the preceding section is thought-provoking. The fact that such obvious and natural principles so easily lead to

[9] A reader interested in this issue may consult an instructive exposition in Copi [1954], Appendix A.

consequences which seem artificial or even false (e.g., that every witch is the devil's accomplice) sheds much light on relations between natural language arguments and logical calculi. The crux of the matter is the **existential import of names**, i.e., the presupposition that the use of a name implies reference to an existing thing. What does it mean?

Our language develops in the environment of existing things; whenever a name is being used, it is implicit in the very fact of its use that this name stands for something. This is no accident and no arbitrary convention. This is a necessity deeply built in our lives. The alternative strategy would be something like that: first to presuppose that the name in question stands for nothing, i.e. its extension equals the empty class, and next to check if, by chance, not the opposite is the case. In other words, non-existence would be constantly presupposed while existence would require a proof. Imagine a hunter who addresses his companion with the warning "a bear is about to attack us". The existential import of the name 'bear' implies that the companion accepts the consequence "there is a bear (close to us)". Should he not accept that and demand a proof, the bear would quickly devour both hunters.

Survival and development are due to collaboration among humans, and collaboration means taking for granted that our communication does not consist in talking about nothing. This rule of the existential import of names is so universal that it controls both everyday speech about empirically given things and the mathematical discourse concerning abstract constructions. There is just the difference, by no means minor, that in mathematics it is existence that should be proved in each case by providing a suitable construction; in everyday communication the existence of what it refers to is presupposed, and non-existence should be proved, if necessary (therefore we do not trust liars but, at the same time, we do not assume from the start that everyone is a liar).

In such a framework, the above quoted statement about witches should be regarded as false since it would imply that witches do exist — in the virtue of the existential import of the name 'witch'. And if a statement implies anything false, it has to be false itself. On the other hand, the proof of its truth is flawless on the basis of some obviously true premises.

The alleged paradox dissolves if we consider the role of logical calculus in the analysis and evaluation of arguments. A calculus should supply means to resolve a compound expression into constituents which are relevant to the validity of the argument in question. The calculus of classes makes it possible to distinguish the negative existential constituent *there is no A not being B* in the universal affirmative statements. Whether such a sentence does or does not include the positive existential constituent *there is an A* should be decided for each concrete context to which one applies one's logical analysis. For the purpose of rendering traditional syllogistic in a mathematical form it was advisable to admit the option without the positive existential component. In this way the class of syllogistic theorems has been divided into those which are provable under this **weak interpretation** of universal statements and those which require a **strong interpretation**, that is including the assertion that the grammatical subject is not empty, i.e., having the existential import. Whenever the weak interpretation does not suit our understanding of a concrete context, we are free to choose the strong one. Aristotle himself preferred the latter route, and his followers contributed to a better understanding of his intentions owing to the analysis outlined above.

**2.2.** Let us consider other examples of applying the calculus of classes to the analysis of those arguments which involve general statements. Two particular statements have been said to be **subaltern** to the respective universal statements and **sub-contrary** to each other while two universal statements are said to be **contrary** to each other; there is also the relation of being **contradictory** to each other discussed earlier (see above 1.2 in this chapter).

A subaltern statement is logically entailed by the respective universal statement only under the strong interpretation of the latter. Suppose that a naive reasoner is not aware of the difference between the two interpretations. Then he might be puzzled by an argument resorting to the **subalternation rule** which claims: *from any true universal you should infer the respective particular statement.* Our reasoner is liable to (recklessly) acknowledge the truth of

(a) Every witch is a witch.

For any thing is identical with itself, and then on the basis of subalternation he is bound to acknowledge that
(b) There is a witch which is a witch,
and so to ascertain the existence of at least one witch. Even worse, from the universal negative statement
(c) No witch is a real entity
he should infer
(d) There is a witch which is not a real entity.

Such puzzles can easily be solved after translating general statements into their class-theoretical counterparts. It is enough to observe that (a) is true only under the weak interpretation, that is one to the effect $A \cdot -A = 0$; in fact, there are no members shared by the classes $A$ and its complement $-A$, also in the case when $A$ is empty. However, the truth due to the emptiness of $A$ gives us no reason to infer the non-emptiness of $A$.

If, on the other hand, we take a universal statement in the strong interpretation, then the existence of an object belonging to the class in question is granted from the very start. We deal then with the compound assertion
(1*) $A \neq 0$ and $A \cdot -B = 0$
as the basis of a reasoning which leads to
(3) $A \cdot B \neq 0$,
that is to the corresponding particular statement (cf. the list of statements in Subsection 1.2). If $A$ is not empty, then (one has to conclude) also $B$ is non-empty; were it empty, its complement mentioned in (1*) would be non-empty, and then together with the non-empty $A$ it could not yield the empty class referred to in equation (1*).

A rhetorical moral which follows from this discussion runs as follows: before we use a universal statement in an argument, we should check if there are reasons to claim the non-emptiness of the subject. If the existence of the entities referred to is neither postulated nor proved in the theory in question, then the reasoner should apply a suitable procedure to prove existence. An everlasting paradigm of such procedures is found in Euclid. They deserve careful examining but before it is made we should enlarge our repertoire of means of expression in a formalized language. In the above proof of the law of subalternation there appear logical

terms which do not belong to the language of the theory of classes, as 'and', 'not' and 'if...then'; they are borrowed from ordinary English as is also 'either...or' being usual in mathematical and other arguments. Their formalization in an exact logical language should grant more rigour to arguments; at the same time it sheds much light upon the structure of arguments in everyday languages. It is why the issue requires a systematic treatment which will be the subject of the next chapter.

However, before we resume the discussion of those calculi which are particularly suitable for analysing arguments, it is in order to point out some significant limitations of the calculus approach. This is meant not to decrease the importance of logical calculi but rather to use them in the role of a filter which should distinguish computable and non-computable constituents of extra-mathematical arguments.

## 3. What names stand for: an exercise in Plato

**3.1.** In the preceding section we dared a risky logical enterprise of coordinating some English contexts, such as those involving 'every' and 'some', with certain operations of the calculus of classes. The risk, which we share with the inventors of this calculus themselves, consists in choosing each of these words from among many with similar functions: do we consider each of them to represent a whole category of English words which are interchangeable with each other (e.g., 'some' might be replaced by 'a(n)', 'at least one', 'all', etc.), or do we give them a specific meaning of their own? In the latter option we restrict the applicability of logical calculus under consideration to those words which have been coordinated with operations of that calculus, and so the calculus proves of little use to render the wealth of argument forms in natural languages. On the other hand, when following the former option, we have to examine that wealth and so face all the vagueness of natural-language expressions and the resulting arbitrariness of interpretation; however, it is that option which has to be chosen if logic is to serve any rhetorical purpose.

In particular, it is the articles *a* and *the* which deserve most careful examination as being the most frequent function words in

the use of English and similar languages. An intriguing problem that must be attacked at the very start results from the fact that the same indefinite article 'a' in some contexts is used as equivalent to 'every' while in others as equivalent to 'some'.

Peculiarities of the definite article are still more thought-provoking, for sometimes it is used to express individuality and sometimes to express generality, and the latter even in a specially strong sense which amounts to claiming the existence of general objects, or universals. A full spectrum of uses, including the last mentioned extreme, can be best found in Plato, and it is why some texts taken from his *Republic* will be used for our logico-linguistic exercises.[10]

The interpretation suggested in this discussion does not pretend to render Plato's original thought, for such a study should take into account the original text as obeying specific rules for the Greek articles. It is rather to be regarded as the discussion of an English text which, owing to its relationship to Plato, provides us with an ample spectrum of uses of articles. In order to tell each of these uses from the others, I suggest some terms invented *ad hoc* instead of distinguishing them by numbers (as is usual in dictionaries of English). These terms allude to the mediaeval theory of suppositions, which is particularly suitable for such discussion since the logicians using Latin, a language without articles, could not even have had the possibility of resorting to the systematization provided by dictionaries. It is why they have invented the technical term *suppositio* — not in the sense of a conjecture, guess, etc. but in the sense derived from the context: *(nomen) supponit pro (aliquo)*, i.e., a name *stands for* a thing. It is this sense in which the English counterpart **supposition** is to be employed in the present section. As for the adjectives distinguishing kinds of supposition, my terminological inventions are somehow inspired by mediaeval Latin terminology but do not tend to follow it; they are devised purely for the purpose of the present discussion.

[10] All the texts are taken from the English translation by B. Jovett, *The Dialogues of Plato*, vol. 3, *The Republic*, 3rd edition, Clarendon Press, Oxford 1892. The numbers in parentheses indicate the book and the passage, the latter according to a standard numbering (Ed. Steph.) applied by Jovett.

I shall distinguish two suppositions which can be expressed by the indefinite article, namely *member supposition* and *class supposition*, and two expressed by the definite article, namely *i*ndividual supposition and *eidetic supposition*, all that being discussed with regard to the problem of the existential import of names as raised in the preceding section.

**3.2. Member supposition** corresponds to that use of the indefinite article which consists in using it as a function word before nouns in the singular form (apart from proper names) and mass nouns when the individual in question is undetermined, unidentified, or unspecified, esp., when the individual is being first mentioned or called to notice; for example, 'there was a tree in the field', or 'a man walked past him'. This grammatical characterization should be complemented by a condition which we owe to the calculus of classes.

Let 'some' in the sentence 'some dogs are philosophers' be replaced by the indefinite article, thus yielding 'a dog is a philosopher'. Is this new sentence equivalent to the former, or not? If one answers in the affirmative, then one takes 'a dog' in **member supposition**, i.e., one has in mind an individual member (or more of them) of the class of dogs. If one answers in the negative, then one resorts to another supposition (to be discussed later). To prove this statement with regard to the member supposition, it is enough to prove the existence of at least one dog being a philosopher, while the other supposition will require more than that; hence in arguing one should be aware of which supposition comes in question. The following passage of *Republic* suggests a criterion to recognize an occurrence of member supposition. Socrates intends to prove that the watchmen in the State should be wise men because watching involves distinguishing between the friendly and the hostile individuals, and that requires a knowledge characteristic of philosophers; the discussion starts with a familiar experience concerning dogs recalled by Socrates (the first speaker in the dialogue quoted below) and successively being confirmed by Adeimantus (ii, 376).

> — A dog, whenever he sees a stranger, is angry; when an acquaintance, he welcomes him, although the one has never done any harm to him, nor the other any good. Did this never strike you as curious?

— The matter never struck me before; but I quite recognize the truth of your remark.
— And surely this instinct of the dog is very charming; – your dog is a true philosopher.
— Why?
— Why, because he distinguishes the face of a friend and of an enemy only by the criterion of knowing and not knowing. And must not AN animal be A lover of learning who determines what he likes and dislikes by the test of knowledge and ignorance?
— Most assuredly.

There are at least two criteria to ascertain that it is the member supposition which comes into play in the above passage. The first of them appears at the very start. It is the temporal setting of the argument, namely the dog in question 'sees a stranger', hence it must be a concrete individual since only the objects of that category can perform such activities.

The other criterion consists in the validity of replacing *a* by *the* in a suitable moment. Such a moment comes when the listener is already acquainted with the object referred to which was unknown to him when the story started. In our story this occurs in the statement ending with the conclusion 'your dog is a true philosopher'. In this context it is clear that people talk about a concrete individual dog, that owned by Adeimantus, hence that use of *the* (which is to be called individual supposition); whenever we deal with such a sequence of using first the indefinite and next the definite article (in individual supposition), the former occurs in the member supposition.

The term 'member' in the role of adjective should indicate that the individual in question is not considered as an individual but as a representative of that class to which the general name, as 'dog' refers to. The statement of the form 'a dog is a philosopher' is true only if there exists a dog, hence the name being its subject must have the existential import. Were there no dogs in the world, this would have made this statement false (obviously, there is another condition of truth, namely that a member of the class of dogs be a member of the class of philosophers; this, however, is irrelevant to the present problem).

**3.3. Class supposition** corresponds to what can also be expressed with the help of the words like *any* and *each* preceding a general

name which is followed by a restrictive modifier, for instance 'a man guilty of kidnapping wins scant sympathy', or 'a man who is sick cannot work well'. This definition, due to grammarians, rightly hints at a restrictive modifier as characteristic of such suppositions, e.g., the modifier 'who is guilty of kidnapping' and 'who is sick' in the above examples.

However, a logician has a reason to ask whether the general name following the indefinite article might not be regarded as a restrictive modifier in some constructions like those discussed above. Consider the following statement made by Plato: 'A State was thought by us to be just when the three classes in the State severally did their own business' (iv, 435). Disregarding unnecessary elements, let us reduce this statement to the following form: *A State is just when the three classes severally do their business.*

There is no restrictive modifier to follow the subject 'a state', nevertheless the statement should be interpreted as universal since it is thought as listing a necessary condition in the definition of a just state (as is obvious with the context of this sentence). The perverse idea suggested by this example is to the effect that the subject may prove to be its own restrictive modifier. To see that let us paraphrase the considered definition as follows: *A system being a perfect State is one in which the three classes severally do their business* (viii, 546).

The trick consists in finding a dummy subject, here 'system', which is always possible (in any case it may be a universal dummy subject like 'thing') so that the old subject becomes a restrictive modifier. Therefore, for some contexts it may be difficult to decide whether we deal with a construction characteristic of member supposition or with a construction admitting a hidden modifier. The latter can be termed a **class supposition** on account of its natural translatability into a statement concerning classes, as in those previously listed examples: 'the class of guilty men is included in the class of those who win scant sympathy', and 'the class of sick men is included in the class of those who cannot work well'. The example of *Republic* would now run: *the class of just states is included in that of systems in which the three categories of citizens severally do their business.*

The suppositions under consideration contrast with one another with regard to existential import. Member supposition grants existential import to a name while class supposition does not. Imagine that there is nowhere in the world a State designed by Plato. This does not refute the truth of his definition; it just means that there are no perfect states deprived of those three categories (i.e., philosophers as rulers, and guardians, and artisans), each of them flawlessly doing its own business. Even if this ideal does not materialize, the Platonian idea of State, as expressed in the above statement, may remain true.

The distinction between member supposition and class supposition starts to vanish when we enter a realm of entities which are indistinguishable from one another as are, for instance, geometrical points. A dog which sees a stranger (to make use of the quoted Socrates' story) is different from a dog which in the moment does not see any stranger, and this temporal difference makes the said dogs different from one another. Hence in such a context the indefinite article makes the name following it stand for individual members of a class, and not for that class itself. However, what about members of a class which are not liable to bear any individual features? Then it is indifferent whether one speaks of one member or of all members; whatever is being said about one of them is true about all the rest. This case is worth a special study which may result in a revision of the grammatical doctrine that the indefinite article marks class supposition only when there appears a restrictive quantifier (to hint at a common property which, so to speak, cancels individual differences).

To suggest a starting point for such a possible study, let me quote a formulation of general theorems in the example of Euclid's Theorem 14 of Book I. It runs as follows:

> If, at *a* point in *a* straight line, two other straight lines, on the opposite sides of it, make the adjacent angles together equal to two right angles, these two straight lines shall be in one and the same straight line.

Other indefinite articles of the same function are hidden in the plural forms as '(*any*) two right angles', etc.

The above feature of such a mixed (so to speak) member-class supposition proves especially interesting from the logico-rhetorical point of view, namely that the error in arguing which consists

in jumping to a hasty generalization may stem from a wrongly presupposed indiscernibility of members of a certain class. For instance, one jumps to the conclusion 'a person who is Jewish has enormous mathematical abilities' — where the article *a* produces a class supposition (to refer to the class of Jewish persons), and 'who is Jewish' is a restrictive modifier — on the basis of the premise 'a person who is Jewish has enormous mathematical abilities', where *a* is taken in member supposition to mean 'there is a person who is Jewish with enormous mathematical abilities'. If there acts in the given mind an unconscious assumption that all Jews are alike in mental qualities (i.e., indiscernible in some respect), it should strengthen the tendency to mistake one of these suppositions for the other. Supposedly, such errors are more likely to be committed by those persons in whose brains there are relatively weak connections between the zone of verbal activities and the zone in which representations of real things are recorded; such brains are more liable to be misled by purely verbal similarities or identities, belonging to what Francis Bacon called *idola fori*.

**3.4.** There is a remarkable parallelism between two suppositions corresponding to the indefinite article and those corresponding to the definite article. Two functions of the definite article which are logically significant are the following. The definite article is used: (i) for mentioning a particular thing either because we already know which one is being talked about or because only one exists; (ii) before a noun in the singular to make it general, e.g. 'the lion is a wild animal' (=lions are wild animals), 'the computer has revolutionized office work'.

The difference between (i) and member supposition consists in the kind of knowledge possessed by a speaker: whether he knows the thing in its individuality, as in case (i), or only as a member of a definite class; yet, these different cognitive situations involve an individual. Let this feature be rendered by the term **individual supposition**; in member supposition the elements of a class are not recognized in their individual traits, yet they are referred to as 'schematic' individuals, so to speak, and in this sense the suggested terminological choice proves justified.

Individual supposition occurs in the English version of *Republic* in the way conforming to general rules which hold for English; there

is nothing specific to distinguish it from other texts or authors. New issues brought up by *Republic* are related to usage (ii), being analogous to class supposition because of the feature of generality. Let it be called **eidetic supposition** in the sense of the term to be explained later in a context of Plato's thought.

There is a passage about the perfect shoemaker to prepare discussion about the perfect guardian. *The shoemaker was not allowed by us to be a weaver, or a builder — in order that we might have our shoes well made; but to him and to every other worker was assigned one work for which he was by nature fitted.* (Book ii, 374). Now let us imagine a botcher who is neither by nature fitted nor sufficiently trained to make good shoes. Does he fulfil the notion expressed in the general supposition '*the* shoemaker'? There are no such doubts as far as either member supposition or class supposition is concerned. It makes sense if one says that he knows a shoemaker who is a botcher (member supposition), or if one says something like this: a shoemaker should pay taxes as do all craftsmen. In the latter context, the term in question appears in class supposition, as the phrase refers to each one belonging to the class of shoemakers.

The same supposition, i.e., that in which a name stands for a class, attaches to the name 'a shoemaker' in the following statement:

[CS] *A shoemaker, as any other person, may prove unfit for his occupation.*

Such sympathetic understanding of human weakness ranges over the class of all people, hence all shoemakers, too. This universality is due to the modal word *may*: whatever actually happens to same persons of some class may happen to other members of the same class. Would this justify inference from class supposition to general supposition? Let us listen to the 'tone' of the following statement which results from CS by replacing *a* with *the*:

[ES] *The shoemaker, as any other person, may prove unfit for his occupation.*

The person talked about in ES is no Platonian shoemaker, for the latter *ex definitione*, hence necessarily, has occupational fitness which does not necessarily attach to the former. The 'tone' in which one utters sentences like ES is that of praising perfection. It

is the tone penetrating the whole text of *Republic* as dealing with the perfect State and its perfect members.

**3.5.** The supposition produced by the definite article in its generalizing function deserves to be called **eidetic supposition** since it deals with an ideal entity; the Greek terms *ειδος* and *ιδεα* are synonymous, and the former is more fitting to create a new technical term since the latter has already too many senses in ordinary and philosophical English. Have we, one may ask, to follow Plato in this respect, and thus to commit ourselves to his controversial philosophical assumptions? It suffices to answer that deeds should be judged by their fruits. In the present logico-rhetorical framework, the chapters concerning definitions should prove how fruitful the adopted Platonian approach is.

Here are some further examples of eidetic supposition.

*In the human soul there is a better and also a worse principle; and when the better has the worse under control, then a man is said to be master of himself.* (iv, 431). In this short passage there appears member supposition with conspicuous existential import ('there is a better principle', etc.), anaphoric use of the definite article ('the better', 'the worse'), class supposition ('a man in which the better has the worse under control'), and against this background the clearly distinguishable eidetic supposition *the human soul*. To hint at such generality as is attached to this principle, Plato needs more than class supposition; he must resort to eidetic supposition because the said properties of soul are not only general but also essential and pertaining to the ideal of soul.

*And so of the individual; we may assume that he has the same three principles in his soul which are found in the State.* (iv, 435). The same thought is developed in the following passage. *Such is the good and true City or State, and the good and true man is of the same pattern; and if this is right, every other is wrong; and the evil is one which affects not only the ordering of the State, but also the regulation of the individual soul.* (v, 449). Here Plato seems to have a feeling that he touches upon the very essence both of the soul and the state, that is the structure consisting of three elements which should act in harmony if the soul or the state in question is to be *true*. Thus eidetic supposition has nothing to do

with appearances; in the sphere of appearances generality may be explained through class supposition, but in the realm of genuine things only eidetic supposition does justice to genuine generality (i.e., not accidental but founded in general objects). Obviously, it is another way of saying that eidetic supposition deals with ideals since — in the Platonian perspective — only ideal things are real things.

Plato's answer to the question of how to conceive generality may be questioned but cannot be disregarded. It will contribute to the discussion of generality in chapters to follow, one dealing with reasoning (Seven) and one dealing with defining (Eight). The way to this discussion leads through the presentation of those logical theories which were lucky to become the standard of modern logic (while the theory of general names is found at its margin), namely the truth-functional calculus and the predicate calculus.

As I end this exercise in Plato, let me address the contemporary philosophers from the noble family of Minimalists (as are empiricists, nominalists, behaviorists, etc.). They are very serious persons who take any Platonic inclinations as a sign of mental frivolity or even deviation. They should be asked to compare the world of physical solids with the world of geometrical objects, and to account for semantic differences between statements describing these two realms. First let them answer if they recognize such differences, and if do, let them next try to account for them in a logico-linguistic theory free of any Platonic bias. This should be a genuine contribution to the theory of definition. This theory heavily draws on fundamental logical calculi, and it is why we need first to discuss them. In will be done in the natural order, first truth-functional logic and then predicate logic.