CHAPTER FIVE

The Truth-Functional Calculus and the Ordinary Use of Connectives

1. The functional approach to logic

1.1. It was mentioned in Chapter Four (Subsec. 2.2) that in demonstrations of class-theoretical theorems we need some **logical connectives**, which do not belong to the language of the theory of classes, namely such words as 'not', 'and', 'either ... or', 'if ... then', etc. That they appear in every reasoning, and that they form the subject-matter of a logical theory (to be discussed in this chapter) is now a fact which is obvious to every educated person. However, it was far from obvious for many centuries, and even less obvious was the idea that syllogistic, e.g., in its class-theoretical version, should belong to a uniform logical system together with the theory of logical connectives. The latter was somehow anticipated by Stoic logic and by the mediaeval treatises *De consequentiis* but an attempt was never made at unifying these two branches of logic, until the turning point made by the German mathematician, Gottlob Frege (1848–1925).

The unifying idea, one that pervades the whole of mathematics and the whole of modern logic, is the concept of **function**. Though practically used in arithmetic, algebra, analysis, etc., from the very beginnings of these disciplines, its first theoretical formulation did not appear until 1749 when Leonard Euler (1707–1783) explained a *function* as a *variable quantity that is dependent upon another variable quantity.* For many purposes such a roughly stated definition was sufficient, but the further development of mathematics brought a more general and abstract concept of function which makes use of class-theoretical notions.[1]

This abstract notion is not restricted to dependence of quantities, e.g., numbers, but it comprises any fact of the **correspondence** between elements of a set, say A, and elements of another set, say V. Let it be so that to each element of A by the correspondence

$$v = \varphi(a)$$

there is assigned exactly (not more, and not less than) one element of V. Then that element v_i of V which is assigned to (one or more) elements a_1, a_2,...,a_n of A is said to be the **value** of the function in question for the **arguments** a_1, a_2,...,a_n. In other words, the function φ is defined on the set A, called the set of its arguments, or its **domain of definition**, and ranges over the set V of its values, or its **range**. For example, the relation of *having a father* assigns exactly one individual to one or more individuals; in this example the domain of definition and the range are identical (e.g., the class of humans). These may also happen to be different; for instance, exactly one ticket corresponds to a group of participants (e.g., a family) of a performance (thus the domain is a class of humans, and the range, a class of documents).

1.2. In some contexts the term 'operation' proves more convenient than 'function', esp. when referring to those functions which one used to call operations from the very beginning of one's school instruction. Such are arithmetical operations which perspicuously exemplify the concept of function as a correspondence: in the operation of division, for instance, to each two elements of the domain of natural numbers there is coordinated exactly one element of the range of rational numbers.

The unification of logic on the basis of the concept *function* has been possible owing to Frege's idea that there are functions for which both the domain and the range equal the two-element class

[1] Instead of the term 'class-theoretical' one often uses the term 'set-theoretical' in contexts like the present one. The theory of classes is regarded as the elementary and basic part of set theory, the latter dealing with the issues of infinite sets. Hence whatever is class-theoretical is also set-theoretical, but for the present purposes I prefer the former term as connected with the historical development narrated in the previous chapter.

of truth-values. The term **truth-value** is to denote two abstract objects, one of them called **truth**, the other **falsehood**. The next bold step was to look at sentential connectives as connecting just truth-values, and at the same time to disregard the other aspects of a sentence, as its meaning, its form, etc. This step was to some extent prepared by the development of the calculus of classes with which Frege was fully familiar.[2]

To notice the continuity (which does not yet diminish the surprising novelty of Frege's insights) let us recall the notions of the empty class and the universal class, and of operations on classes, as discussed in the previous chapter (subsec. 1.3); the operations mentioned there are complementation and intersection. The complement of the empty class amounts to the universal class, and vice versa, which is a trivial common-sense observation, as the empty class and the universal class form together the universal class. In symbols:

$$-0 = 1 \text{ and } -1 = 0;$$

e.g., if one takes the class of plants as universal (1), then (in a universe of discourse so defined) the class of non-plants (-1) equals 0. The intersection operation as applied to these two distinguished classes is defined by the following equations:

$$1 \cdot 1 = 1, \quad 1 \cdot 0 = 0, \quad 0 \cdot 1 = 1, \quad 0 \cdot 0 = 0.$$

Obviously, the elements which the universal class has in common with itself again form the universal class; there are no elements which would be common to the universal class and the empty class, i.e. the operation results in the empty class, etc.

It may be asked why one should dwell on logical facts so elementary that they belong to the basics of general education. However, they deserve mention to support the claim that such basic notions of elementary logic comply with the common mental conduct of people who state arguments in ordinary language. Thus the functional approach which results in those logical notions proves to be fruitful from the rhetorical point of view as well as well. This determines the strategy of subsequent discussion. There are at least two truth-functional operations which undoubtedly accord with

[2] See G. Frege, *Booles rechnende Logik und die Begriffschrift* in Frege [1973], p.180.

some uses of ordinary connectives; this encourages us to define other connectives in terms of those basic ones, and so to examine relations between them and between their ordinary-language counterparts.[3]

1.3. Teachers of formal logic used to pay much attention to the so-called paradox of implication, which consists in a certain discrepancy between the inference rules for the ordinary-language conditional and their truth-functional counterpart. The problem is regarded by some scholars as serious; logical calculi, one claims, should help to check the validity of arguments, and this is possible only if inference rules are identical in ordinary language arguments and in the calculus applied to check them. There is also a problem, though less frequently raised, of the ordinary use of *either ... or* and the meaning of its formal counterpart, called disjunction.

There are in fact some problems, but they result rather from too superficial a philosophy of logic than from an actual conflict between logic and ordinary language. This philosophy is related to what might be called **folk logic**; let this term be patterned on the nowadays fashionable phrase 'folk psychology'.[4] Folk logic involves the principle pertaining to logic itself that *logic should rule the whole of human cognitive conduct* or, at least, the conduct of a philosopher, a scholar, etc. This principle is associated with the development of logic from its very beginnings, from Aristotle's *Organon* up to viewing logic as *Medicina mentis* in the way characteristic of the Cartesian School (cf. Chapter Three, Subsec. 2.2). In modern logic this principle is hardly traceable, but there are traces of it in folk logic inherited from older logical theories. No wonder that they have a hold on some minds, namely those happily

[3] This claim may seem so evident that it should not require any defence. There is, though, the great, even if recently decreasing, influence of "ordinary language philosophy" (which emerged after the Second World War and was centred in Oxford University) which makes it fashionable to disregard formal logic and to base a theory of argument by attending to the structure of ordinary usage alone. This was an understandable reaction to the excessive formalism of the prewar neopositivistic manifesto, but the reaction itself proved also exaggerated, and now the time is ripe for a more balanced approach.

[4] The latter is discussed in Sterelny [1990], the book which is here of interest for some other reasons, too.

rooted in good old traditions, and so liable to overlook something new.

Even if logic taken in the broadest sense, hence without any restricting adjective, should obey the above principle, this certainly does not pertain to *formal* logic, i.e., that consisting of logical calculi. Due to awareness of this, the present discussion distances itself from those who first fall into a trap carefully devised by themselves, and then display a wonderful ingenuity to get out of it. This is why the exposition starts from two logical connectives which combine the feature of being basic, i.e., being such that all the other connectives are definable in terms of them, with the feature of being closest to the meanings of their counterparts in truth-functional logic. Owing to that definability relation, the closeness in meanings should be inherited by the connectives defined in terms of those chosen as basic. This forms the subject matter of the next section.

2. The truth-functional analysis of denial and conjunction

2.1. For a moment we should forget logic and its applications and deal only with an abstract calculus of two objects, one of them being denoted by '1', the other one by '0' (any resemblance of these shapes to symbols known earlier is only accidental). Let the two-element set including 0 and 1 be both the domain of definition and the range of functions we are to define. We are free to create functions of as many arguments as we like, that is one-, two- , three-argument functions, etc. For instance, one can define the three-argument function ψ by which to each ordered triple of 0's and 1's as arguments (the number of such orderings being 2^3 for any triple) there corresponds 0 unless the sequence consists of 0's alone; in this case let the value of the function be 1, as expressed in the formula $\psi(0,0,0) = 1$. As one can define functions of arbitrarily many arguments, there are infinitely many such functions.

For reasons to become apparent later, we take into account only one-argument and two-argument functions. For the former category there are four (2^2) functions to be defined, for the latter — sixteen (4^2), as seen in the following tables; the symbol at the top of a column (a letter in T1, a numerical symbol in T2) can be used to name the function in question.

x	A	B	C	D
1	1	1	0	0
0	1	0	1	0

T1

Before displaying the next table, let us take advantage of this one to discuss the transition from the abstract calculus to its interpretation in logic.

The interpretation consists in making the symbols '1' and '0' stand for abstract objects called *truth-values*, namely the **True** and the **False**, respectively, and then making the function symbols (A, B, C, D) stand for operations on truth-values. Thus function C becomes the operation of **denial**, called also **negation**, which performed on the True yields the False, and performed on the False yields the True. It is not a priori granted that each of such combinatorily obtained functions has an intuitive logical interpretation, or that all interpretations are equally intuitive. In any case, the interpretation of B as **assertion** — in a similar sense as saying 'ditto', i.e., confirming something having been already said — is fully intuitive. It is more difficult to find an intuitive realization for A and D, but these functions may have some technical uses.

As for the functions B and C, it is the latter, i.e., the denial, which has enormous importance for formulating and scrutinizing arguments. Its behaviour is exactly like the behaviour of the ordinary expressions which are attached to a single statement, either prefixed to the whole statement, as in the phrases 'it is not true that', 'it is not the case that', etc., or prefixed to the predicate, as in the word 'not'. In each case the True is transformed into the False, and vice versa. Thus the construction of Table T1, though made entirely a priori and listing mere possibilities, supplies us with a logical means which is omnipresent in actual arguments. A similar experience awaits us in the case of two-argument functions.

2.2. Each argument in a pair (x, y) takes either the entity 0 or the entity 1 as its value, and this yields four combinations, viz., $(1,1), (1,0), (0,1), (0,0)$. For each pair of argument values there is exactly one value of the given function, that is either 1 or 0. Four pairs require four such coordinations to be made in 16 possible ways, as shown in the following table.

x y	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1 1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1 0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
0 1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
0 0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

T2

When we take the symbols '1' and '0' as standing for truth-values, it proves that some functions listed in the above table can be interpreted as English connectives. In particular, function 8 renders the behaviour of 'and'. When 'and' connects two statements both referring to the True (i.e., simply, being true) the connection results in the True, otherwise it results in the False. This function is termed **conjunction**.

There is a noteworthy feature of the ordinary use of 'and' which cannot be rendered in the abstract theory of truth-values. I suggest that this feature be discussed in terms of **praxeology**, i.e., *the general theory of purposeful human action.* Among the basic praxeological categories are those of **efficiency** and **economy** of action (this example hints at a relation between praxeology and decision theory; the latter presupposes as given some fundamental concepts worked out by the former).[5] The concepts of praxeology are deliberately so general than they can have instantiating applications in various domains of human action, e.g., in economic activities as studied by Ludwig von Mises [1949]. Certainly, the domain of human *communication* is an important field to be studied with the help of praxeological categories, such as purposefulness, efficiency, economy, rationality, utility, etc. Some of these concepts are to be adopted to examine the use of 'and' as well as other connectives.

[5] The term *praxeology* was first used in 1890 by Alfred Espinas in his article 'Les origines de la technologie', *Revue Philosophique* 15, year 30, 114-115, and his book published in Paris 1897, with the same title. The idea of praxeology forms the basis of the economic theories of Ludwig von Mises as presented, e.g., in his *Human Action* [1949]. The Polish philosopher and logician Tadeusz Kotarbiński (1886-1981) pioneered a philosophical theory of human action under the same term; see, e.g., his *Praxiology* [1965].

Anyone uttering a compound statement of the form p and q (the letters indicate the places to be filled by constituent statements) does so for a certain purpose, and this implies that both constituents carry useful information and that there should be a nexus between their meanings. The latter condition prevents a situation in which two messages, both useful for the addressee but pertaining to very different subjects, would be combined in one conjunctive sentence. Compare, for example, the following utterances:

(a) 'Here starts a slippery section of the road; *anyway*, the landscape on its sides is very attractive';

(b) 'Here starts a slippery section of the road *and* the landscape on its sides is very attractive'.

The utterance (a) may be so interpreted that there is none or only a slight connection between the components statements. The speaker is aware that in the second statement he allows himself to express his casual association or remembering, not necessarily justified by its usefulness for the listener, while the first statement is a warning to express a care; this awareness of the lack of a closer connection is hinted through the use of 'anyway'. The use of 'and' in (b) suggests a stronger connection. It is likely to be interpreted as follows. There is an additional reason to be cautious, namely the danger of being distracted by the beauty of the landscape in circumstances requiring special attention, i.e., when driving a car. Were it not so, were the second member of the conjunction added only as a report on the speaker's esthetic impressions, this construction could then be charged with committing the praxeological error of the lack of what I shall call **communicative relevance**; this kind of relevance consists in combining two or more messages into one syntactic construction, e.g., conjunction, only if there is a nexus between their meanings. What nexus is required, is defined by the set of conventions governing the ordinary usage of expressions.[6]

[6] I take the convenient term *kommunikative Relevanz* from Posner [1972], a book in which this concept plays an important role. The present use of this term does not match its elaborated definition in the said book, it is rather introduced *ad hoc*, to take advantage of its suggestive associations.

The conjunctive construction (b) in the above example lacks communicative relevance in the case of being so interpreted that there is no nexus of content or a purpose between the messages conveyed by its components. Such a lack of nexus belongs to the category of **praxeological fallacies in communication** (other fallacies of this kind are to be discussed in what follows). These should be distinguished from logical fallacies in arguments. The latter are definable on the basis of a logical calculus as is to be shown below.

2.3. To assess the validity of an argument, that is its being free from any logical fallacy, we need the concept of **logical truth** which, when applied to a logical formula, can be roughly defined as a *formula true by virtue of meanings of its logical terms alone* (a more precise definition is to be given later with the help of the truth-functional calculus). Another name for logical truth is **tautology**.

Let it be noted that the latter term has its origin in ancient rhetoric. In its Greek form $\tau\alpha\upsilon\tau o\lambda o\gamma\iota\alpha$ it appeared in the Roman rhetorical writer Dionisios of Halikarnas (at the time of Augustus) to mean a pleonasm, i.e., a redundant phrase that does not convey any new information with respect to what has been said previously. The assimilation of this term to logic is due to Ludwig Wittgenstein (1889–1951) who in his *Tractatus* defined tautology as a sentence that admits of any possible state of affairs (*lässt jede mögliche Sache zu*, item 4.462), hence does not provide us with any information about the actual world.

The trait of being a tautology is conspicuous, e.g., in the denial of contradiction, that is in saying that two contradictory statements cannot both be true. Truth-functional logic offers us a calculatory method of checking whether a formula does or does not belong to tautologies. Let it be shown in the mentioned example referred to as the **law of (the denial of) contradiction** (the part in the parentheses is usually omitted).

In the formulas of truth-functional logic, the English words 'not' and 'and' are replaced by some artificial symbols; let us accept the symbolic notation in which denial is rendered by the sign $\neg$, and conjunction by the sign $\wedge$.[7] To perform calculation we need

[7] A review of logical notations (each of them having a heuristic merit) can be found in *Logic* [1981], article 'Logic, modern' by W. Marciszewski, item 3. 8.

pertinent parts of Tables T1 and T2. Let these parts be singled out as TN for denial (negation) and TC for conjunction; because of their use in calculating the truth of tautologies such devices are called **truth-tables**.

TN

p	$\neg p$
1	0
0	1

TC

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

With the help of these truth-tables we calculate the truth of the law of contradiction

CL $\quad \neg(p \wedge \neg p)$,

where p is a **sentential variable** to represent any statement whatever, in the following way.

In the case of $p = 1$, after substituting 1 for p, we have the following equalities:

$\neg(1 \wedge \neg 1) \ = \ \neg(1 \wedge 0) \ = \ \neg 0 = 1.$

In the case of $p = 0$ there are the equalities:

$\neg(0 \wedge \neg 0) \ = \ \neg(0 \wedge 1) \ = \ \neg 0 = 1.$

Thus CL proves true for any truth-value of a statement which would fill up the place marked by the letter p. The existence of such forever-true assertions, the truth of which is provable in a purely calculatory way, is a real philosophical discovery. It was anticipated by Leibniz's notion of a statement being true in all possible worlds. In fact, at the place of p in CL there may occur any sentence whatever, hence any sentence describing any possible world (e.g, "all humans have spherical bodies"), and the assertion like CL will always prove true.

Let me further exemplify the notion of tautology by a version of the theorem termed the **law of simplification**.

SL $\quad \neg((p \wedge q) \wedge \neg p), \quad \neg((p \wedge q) \wedge \neg q)$

As there appear two sentential variables, each being substitutable either by 1 or by 0, there are four possible substitutions, namely:

p	q	substit.
1	1	*i*
1	0	*ii*
0	1	*iii*
0	0	*iv*

Table of substitutions

For substitution i, one obtains

$\neg((1 \wedge 1) \wedge \neg 1) \;=\; \neg((1 \wedge 1) \wedge 0) \;=\; \neg(1 \wedge 0) \;=\; \neg 0 \;=\; 1.$

A simple check for substitutions *ii*, *iii* and *iv* again shows that the truth-value of SL equals 1, thus SL proves true in every state of affairs ('in all possible worlds'), i.e., it proves to be a tautology.

2.4. The above result sheds light on the difference between logical validity and praxeological correctness in communication, as discussed above (Subsec. 2.2). The lack of communicative relevance (a praxeological fallacy) in example (b) does not do any harm to the logical validity of argument which might be based on SL. Let us recall the statement in question:

(b) 'Here starts a slippery section of the road *and* the landscape on its sides is very attractive'.

By virtue of SL it can rightly be inferred from (b), assumed as true, that its first component as well as the second one has to be true. For SL, due to the denial sign prefixing the formula, rejects the case that (b) would be true whilst its first constituent would be not-true; hence, provided the truth of (b) one has to acknowledge the truth of its first constituent; the same deduction applies to the second constituent.

Thus the argument proves logically valid even if its first premise, viz., the conjunction, lacks communicative relevance. This clearly shows that a critical examination of an argument should be split into two parts, one concerning the logical aspect of the argument, the other its praxeological aspect. However, the latter aspect happens to be regarded as logical by folk logic which tends to subsume some other problems of correctness under logic, including those termed above as praxeological.

This distinction should remove headaches occurring to those teachers of logic who give examples like (b) and then face the objection that there is (in the sense of folk logic) something illogical in them. This prophylactic measure taken to prevent misunderstandings and objections will prove even more useful when dealing with other logical connectives, those to be discussed in the next section.

3. The truth-functional analysis of disjunction

3.1. *Paul is either a man of genius or is crazy* (example 1). Such a disjunctive statement may be heard when *someone's ideas go far beyond either common views or an average capacity of understanding* (example 2). Let this opinion be examined as an example of the ordinary use of disjunction.

Let us imagine that the connective *either ... or* has somehow disappeared from English. Should it mean that we no longer be able to express disjunctive thoughts (like those in examples 1 and 2 above)? Fortunately, the answer is optimistic. The apparently scant set of connectives reduced to denial and conjunction proves, in fact, able to express disjunction and some other operations, though at the cost of a more complex or cumbersome linguistic structures.

Example 1 can be translated into that impoverished language as follows:
It is not the case that Paul is no man of genius and that Paul is not crazy.
In the language of the truth-functional calculus, when abbreviating the first component of the denied conjunction as G and the second as C, one renders the statement under consideration in the following form:

Ex1 $\neg(\neg G \wedge \neg C)$.

Now, using tables TN and TC we can calculate which truth-values of the component statements make the whole statement (i.e., the denied conjunction of denials) true and which make it false. For instance, when $G = 1$ and $C = 0$, the calculation runs as follows:

$\neg(\neg 1 \wedge \neg 0) = \neg(0 \wedge 1) = \neg 0 = 1.$

After having exhausted all the four substitutions, we obtain the following table.

G	C	$\neg(\neg G \wedge \neg C$
1	1	1
1	0	1
0	1	1
0	0	0

TD*

3.2. Does the above result agree with our instinctive understanding of the connective *either ... or*? Suppose that Paul is not a man of genius and is not crazy, as listed in the last line of TD*. Then the statement discussed as an example, according to the ordinary understanding of *either ... or*, becomes false, as is stated in the table, too. The same ordinary understanding confirms the results of calculation in those lines in which one of the statements is true while the other is false.

There may be a problem about the first line in which both component statements are taken to be true. Suppose that Paul is both a genius and is crazy, which would even accord with some theories of genius (e.g., that developed by Thomas Mann in his *Doctor Faustus*). One might then ask if the speaker had a good reason to use the form of disjunction. However, this question is praxeological, and not logical. Here the rule of communicative behaviour seems to function, in prescribing the following:

> CRD (for Communication Rule for Disjunction): use the disjunction only in those cases in which you do not know which member of the disjunction is true, while you know that one of them is true. Should this rule be completed to the effect that the last clause would read: 'you know that *at least one of them* is true'?

Example 2 (at the start of this section) is meant to shed light on this question.[8] When the present author uses the statement referred to as example 2, he follows the rule in question in the version involving the *at least* proviso. For the sake of convenience of description, let the disjunctive property *to be a man of genius or crazy* be referred

[8] Let the quoted text be repeated for the reader's convenience. It runs as follows. *Paul is either a man of genius or is crazy* (example 1). Such a disjunctive statement may be heard when *someone's ideas go far beyond either common views or an average capacity of understanding* (example 2).

to as D. The present author is not certain whether the average person attributes D to Paul because of the enormous difference of content between Paul's own ideas and his/her own or because of difficulties in understanding Paul's ideas; I can, however, be certain that *at least* one of these circumstances accounts for that opinion concerning Paul, and this means that his statement will also be verified when both circumstances prove to be the case.

Analogously, the view attributing D to Paul would be verified in the case of Paul's being both a man of genius and crazy, provided that those who maintain this view do interpret 'either ... or' in this non-exclusive way, viz., admitting the occurrence of both alternatives, called **inclusive disjunction**. The cautious clause *provided* is used because of the awareness that there is an ambiguity about this connective in English and its counterparts in some other languages. Acting as the author of the statement referred to as example 2, I am able to report on my own interpretation of 'either ... or' occurring in this statement and thus to use this statement as an exemplification of the connective defined in TD*. However, if someone prefers to use 'either ... or' in the way which does not admit of both alternatives, i.e., as **exclusive disjunction**, that interpretation can be rendered in the truth-functional language as well, namely it corresponds to operation 10 in Table T2 (see Subsec. 2.2 above). In some languages there are two words to do the duty for the English 'either ... or'. In Latin *vel* and *aut* correspond to inclusive and exclusive disjunction, respectively.[9]

When using the term 'disjunction' without any adjective, I mean the inclusive sense of disjunction, corresponding to operation 2 of T2, and expressed in terms of denial and conjunction in Table TD* above. Instead of rendering the disjunction adopted in TD*, it is practical to devise a special single symbol for it; let it be '$\vee$' (to suggest the meaning of the Latin *vel*). Here is its truth-table (TD* rewritten as TD).

[9] A reader more interested in this subject should consult Robinson [1978] where section 7.5 deals with the two meanings of 'or'.

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

TD

3.3. The correspondence between the ordinary connective 'either ... or' and its truth-functional counterpart does not exhaust its whole meaning. There are two praxeological features to add something to that meaning. One of them derives from the requirement of communicative relevance, as previously stated for conjunction (in 2.4 above). In the ordinary use of disjunction nobody would say something like this: 'either Newton discovered gravitation or Bill Clinton will succeed in reducing the US debt'. There is no point in uttering such a statement, in spite of its being true (due to the first component stating a historical fact), hence such an utterance would be a praxeological fallacy.

In addition to sharing this feature with conjunction, a disjunctive statement has one feature which is characteristic of it alone. This specific feature is defined in the rule CRD (see Subsec. 3.2 above). According to this rule, a disjunctional statement claimed as true should express the combined state of certainty as to its truth as a whole, and of uncertainty as to the truth of either of its members. In the ordinary use of disjunction, nobody would say something like: 'Bonn is either the capital of Germany or the capital of Cuba' — when everybody, including the speaker himself, knows that the first component is true and the second is false.

What about a situation in which the speaker alone knows which component is true while his audience does not? Suppose the Prime Minister of a State were asked about the date of a national election, to be announced by him, but for the time being wished to conceal the date without telling a lie; therefore he mentions two dates in the disjunctive manner ('either this or that'), one of them being true according to his knowledge, the other one being wrong. The question belongs to what might be called the *ethics of communication*, related to praxeology of communication; this should perhaps be settled by recourse to the precept of telling the whole truth with admissible exceptions to this precept. Whatever ethical solution would be right, from the rhetorical point of view it is advisable to take advantage of logical structures which allow of speaking the truth without imparting too much information.

To state this idea in more general terms, it should be said that the conjunction serves to augment information while the disjunction serves to reduce it. Let us consider a single statement p. It follows from, e.g., $p \wedge q$, and not vice versa, hence the conjunction tells us more than its single component. On the other hand, the disjunction $p \vee q$ follows from the single p (as well as from q), and not vice versa, hence the single component tells us more than the disjunction as a whole; the more components it includes, the less information it conveys. These logical laws provide speakers and writers with an expedient method of dosing information according to their purposes, yet without damaging the truth.

4. The truth-functional analysis of conditionals

4.1. *If one is endowed with instinctive logic, one observes the consequence rules* (example 3). *If I use this statement as a case to be studied, this is due to some inspiration drawn from St. Augustine* (example 4). The statement is closely related to the main point of this essay, it might even be used as an introductory motto; hence, it deserves being examined for both its content and its form. And what about St. Augustine? In his work *De doctrina Christiana* (iv, 3) he made a witty comment on the rules of eloquence, that is certain rhetorical and logical canons. It reads: it is the case that people observe rules because they are eloquent, not that they adopt them to become eloquent. In his own words: *implent quippe regulae, quia sunt eloquentes, non adhibent ut sint eloquentes.*[10] Both Augustine's remark and the tenet of this book emphasize the import of the instinctive skill at arguing. This, in turn, implies that we need a logical theory first in order to better understand our minds, and only then to improve them due to such understanding.

To sum up, example 3 is so chosen that its form should serve as an instance of conditional statements, while its content should help

[10] See *Migne Latinus*, vol. 34, p. 91.

in studying some relations between form and content in conditional assertions. To start with, as in the preceding section, we imagine that the connective 'if ... then ...', (henceforth abbreviated to *if*), has disappeared from the English language. This does not mean that we are unable to express conditional thoughts such as those in examples 3 and 4 above. Again we can use denial and conjunction to express the same thought, or at least that part of it which is crucial for the logical validity of an argument.

Example 3 can be translated into such reduced language as follows:

It is not the case that one is endowed with instinctive logic and does not observe the consequence rules.

Example 4 might take the following form:

It is not the case that I use this statement [...] and this is not due to some inspiration drawn from St. Augustine.

Or, in a freer translation: I use this statement not without some inspiration drawn from Augustine.

Let us scrutinize example 3 with the truth-functional calculus, abbreviating the first component of the denied conjunction as E (for 'Endowed') and the second as B (for 'oBserves'). Then the statement takes the following form:

Ex3 $\neg(E \wedge \neg B)$.

Now, using Tables TN and TC we can calculate which truth-values of the component statements make the whole statement, i.e., the denied conjunction, true and which make it false. For instance, if $E = 1$ and $B = 0$ (see the second row in TC) the result of calculation is as follows:

$\neg(1 \wedge \neg 0) = \neg(1 \wedge 1) = \neg 1 = 0.$

Taking into account one by one the pairs of truth-values listed in the other rows, we ascertain that in all remaining cases the examined function takes 1 as its value. This result is displayed in the following table called TI, where the letter I hints at the term **implication**, which is another name for conditional.

E	B	$\neg(E \wedge \neg B)$
1	1	1
1	0	0
0	1	1
0	0	1

TI*

4.2. Do these results agree with the intuitive understanding of the connective *if* as guided by our logical instinct? Suppose the *one* mentioned in example 3 is identical with someone called Smith who is known to be endowed with instinctive logic and to disobey logical consequence rules; in symbols, (**1**) $E \wedge \neg B$. This instance obviously denies the statement 'If Smith is endowed with instinctive logic, Smith observes the consequence rules', that is (**2**) *if E, then B*. Since **1** is the denial of **2**, it follows that the denial of **1** amounts to **2**; this means that the statements $\neg(E \wedge \neg B)$ and *if E, then B* do state the same fact. Thus at this point our intuitive reasoning agrees with the truth-functional analysis.

Suppose that Smith does not enjoy any instinctive logic, and he observes consequence rules (for communicative relevance 'but' would be better than 'and' in the present sentence, but logically these connectives are equivalent). To express this symbolically:

$\neg E \wedge B$ (see the third row of TI*).

Is, then, the conditional intuitively true? The answer is in the affirmative. Once we have agreed that

if E, then B means the same as $\neg(E \wedge \neg B)$,

we should assert that $\neg E \wedge B$ verifies this conditional as it verifies its equivalent, viz. $\neg(E \wedge \neg B)$. And the latter is verified, since when $\neg E$ is the case, then the statement E must be false; then any conjunction containing E (as is $E \wedge \neg B$) has to be false, hence the denial of such a conjunction as $\neg(E \wedge \neg B)$ proves true.

A similar discussion concerning the remaining rows of TI* leads to ascertaining that that table records the same results which can be obtained through an intuitive reasoning, provided that in each case the accepted supposition *if E then B* is interpreted as in example 3, that is as the denial of the simultaneous occurrence of the antecedent and the denied consequent.

Let this result be summed up by putting TI* into a more general form TI, listing schematic letters p and q instead of the concrete statements E and B and providing a special symbol for the conditional, viz., an arrow, to replace the cumbersome schema of the negated conjunction (as provisionally used in TI*).

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

TI

4.3. The correspondence between the ordinary connective ‘if’ and its truth-functional interpretation in TI does not exhaust the whole meaning of the former. Again, certain praxeological features should be considered to add something to that meaning. One of them derives from the requirement of communicative relevance, as previously stated for conjunction and disjunction (in 2.4 and 3.3, respectively).

This requirement is to the effect that in each conditional there should be a nexus between the content of its antecedent (i.e., the sentence preceding ‘then’) and the content of its consequent. A conditional like ‘if there is snow in mountains, then there are fishes in seas’ has to be true because of the truth of both components but it is neither expected nor recommended to be used in the process of communication, for there is no point in using it.

The above praxeological rule contributes much to the understanding of the present rhetorical point concerning logic. It sheds light on the self-sufficiency of **classical logic**, i.e., the truth-functional calculus and the predicate calculus (as presented in the next chapter), provided that this logic is accompanied by certain praxeological principles to be observed together with its rules. Besides classical logic there are **alternative logics** being proposed as a remedy for some troubles which it is claimed that classical logic is incapable of handling.

It is not the contention of the present argument to challenge the use and merits of alternative logics. I just wish to hint that for rhetorical purposes there are at least two options open, one of which consists in replacing classical logic with some of its alternatives, while the other one consists in using classical logic together with appropriate praxeological principles of communication. The latter option has been chosen in this essay as the one that better introduces us into the main ideas of logic.

Among the alternative logics, there is one which would be a favourite candidate for our rhetorical purposes, were it not for the fact that in doing so we resort to the praxeology of communication. Even the name of this logical theory hints at the problem we now intend to deal with, as its name reads **relevance logic**. Defenders of relevance logic argue in a way similar to my own as sketched above, for instance when saying such things as: "Implication (entailment) is a necessary connection between meanings" (Nelson [1930]). A formalized approach to the problem of how to render this connection through purely logical ways was initiated by Ackermann [1956] who with regard to what he called **rigorous implication** said that it is to express a logical nexus between the A(ntecedent) and the C(onsequent), such that the content of C is part of the content of A, irrespective of the truth-values of A and C.[11]

Ackermann [1956] lists some laws of classical logic which should not hold in any relevant logic. Let me discuss what is regarded as a specially drastic case, namely the law that from contradiction there follows anything, in symbols

$$(p \wedge \neg p) \rightarrow q$$

or, equivalently,

$$p \rightarrow (\neg p \rightarrow q)$$

which bears the name of the famous mediaeval logician Duns Scotus, and which caught the attention of such champions of logic and

[11] There is a tremendous wealth of literature devoted to the logic of relevance. Let me just mention the classical two-volume position by Anderson et al. [1975], [1992], whose first volume is dedicated to W. Ackermann as the founder of this branch of logic. The first attempts to make implication closer to natural reasoning are due to Lewis and Langford [1932]. A very useful overview of the current state of relevant logic, combined with the author's own ideas and results, is found in Read [1988].

mathematics as Bertrand Russell and Henri Poincaré. That attention is recorded in some anecdotes. Poincaré [1924] used to say that a novice in mathematics has troubles in proving theorems only up to the moment in which he unconsciously assumes a contradiction; the rest then goes very smoothly, as he is able to prove anything he wishes, even a truth. Russell, when asked to give an example of how any statement whatever, say that Russell (a renowned atheist) is the Pope, might follow from the self-contradictory statement 5=2+2, suggested that 3 be subtracted from both sides of this supposed equality; it follows that 2=1, thus two different persons, viz. Bertrand Russell and the Pope, form one person; hence Russell is the Pope.

There is nothing about this law that should startle people engaged in arguing, even if they are not so sophisticated in logic as were Russell and Poincaré. If a partner commits a contradiction, then he is expected to demand the acknowledgment of any claim he makes. For, according to his own convenience, he may make use either of one or the other member of the contradiction which, consciously or unconsciously, he has made.

It should be noted that the requirement of communicative relevance in the use of a conditional, as stated in the above quotations related to the logic of relevance, was formulated in a stronger form than was done above in the cases of conjunction and disjunction. For the conditional, it is required not only that its components be concerned with the same subject or domain; there is a yet stronger requirement, viz., that of 'a necessary connexion' or of 'a logical nexus'. This new feature, though, should also be construed in terms of communicative relevance. The use of a conditional is subjected to the rule that it is communicatively relevant only if a logical nexus holds between the antecedent and the consequent.

4.4. The requirement of communicatve relevance with regard to a conditional seems so essential that it may be asked why this problem is disregarded in expositions of truth-functional logic and in comments concerning its applications. It proves, however, that when in arguing one is guided by some suitably selected inference rules concerning the use of conditional, then one is not bothered by any paradoxical consequences. In what follows I shall give an example of such common-sense rules, granting the communicative relevance.

So far, the phrase ‘inference rule’ was used in this essay in an intuitive way, assuming that whoever understands the notion of inference and that of a rule should properly construe the phrase in question. A more systematic treatment, in the sense of listing and commenting some set of inference rules, is to be found in the next chapter. The most elementary introduction to truth-functional logic can be made without formulating inference rules, due to the truth-table method which makes it possible to check validity of any formula without inferring it from other formulas. Hence it is not necessary for truth-functional logic to be constructed and presented in the form of a **deductive system**, that is to say, a set of theorems in which each theorem is either asserted as an axiom, i.e., without a proof, or is derived from the previously accepted theorems with the use of inference rules.

It is possible for truth-functional logic to be constructed as a deductive system, and for some methodological reasons many authors present it in that form, but this is not necessary in the present exposition as focussed on the use of truth-functional connectives in ordinary-language arguments. However, the discussion of conditionals and their praxeological aspects can profit from a comment on a chosen example of inference rules.

The means of inference offered by symbolic logic function either as theorems stating that such-and-such is the case (logical theorems state that for all possible worlds) or as rules concerning operations. Hence in constructing a deductive system one has a range of free choice, limited only by some conditions to be satisfied by the system (as consistency, independence of axioms, completeness, etc.). For logical systems it is possible to be constructed without theorems at all, only in the form of a set of inference rules. Nevertheless, it is useful to take into account both mentioned forms. Forms with particular pertinence to the use of conditionals are discussed in what follows.

Firstly, there is the theorem called **modus ponens** after its old Latin name *modus ponendo ponens* to mean the mode of reasoning which starts from a position, in the sense of an affirmation, and thereby leads to an affirmation (while other modes start from a negation, or lead to a negation, etc.). Here is its truth-functional formulation:

[PP] $((p \to q) \wedge p) \to q$.

The rule which is its counterpart, called either the modus ponens rule or **Detachment Rule** (the latter name will be preferred), runs as follows:

[**DR**] from $\phi \to \psi$ and ϕ infer ψ.

For instance, a politician bases his calculations on asserting the statements of the form:

(i) If the Government creates unemployment, then severe industrial unrest will follow.
(ii) The Government creates unemployment.

Then by the virtue of **DR** the politician predicts severe industrial unrest (which may help him in finding a mode of persuasion).

What additional light can this shed on the praxeological relevance of a conditional (apart from points discussed in 1.3 above)? The use of conditionals in the context of **DR** belongs to the most typical ones. We need conditional statements in order to make inferences guided by that rule. For this purpose one must believe in the truth both of the conditional and of its antecedent. And, as to the former, people (except in some artificially constructed cases) have no other grounds for such belief as their knowledge (or, at least, belief) in a physical, conceptual, or else logical, nexus between the antecedent and the consequent. Only that makes them sure that if the antecedent is the case, then the consequent must be the case, too. Even if people are aware that truth-functional logic allows them to assert a conditional without knowing such a nexus exists, they never do that for this simple reason that the knowledge about a nexus is (as a rule) the sole source of knowledge about the truth of the conditional. Authors who present symbolic logic (if not guided by rhetorical considerations) are not bound to concern themselves with the discrepancy between the truth-functional definition of a conditional and its actual applications. Whether they comment on it or not, people accept conditionals within the range of their cognitive possibilities, and these restrict application to those cases which involve a nexus between the constituents; this is specially evident in the use of the detachment rule.