

CHAPTER EIGHT

Defining, Logic, and Intelligence

1. The ostension procedure as a paradigm of definition

1.1. According to the official doctrine found in textbooks of logic, there are genuine definitions called **normal** (see Section 2 below) in which the term defined and the defining phrase have identical extensions, and there are some deviant sorts which do not meet this identity requirement (as, e.g., a terminological explanation by examples in which the sum of extensions does not match the extension of the term so explained). Among the latter there is the procedure of introducing new terms through ostending exemplary objects which these terms stand for; this procedure is called **ostensive definition**.

This disregarding of ostension by the official doctrine may be explained by taking into account mainly the practice of mathematicians who do not deal with entities capable of being perceived by the senses; at the same time they enjoy the privilege of being able to formulate most precise normal definitions of abstract objects which are constructed by them and therefore are present, so to speak, inside their minds and do not need to be observed from the outside.¹ On the other hand, every normal definition,

¹ This view is certainly right as far the professional practice of mathematicians is concerned. However, some basic mathematical notions which precede a professional treatment of them, such as is the notion of cardinal numbers, and that of natural numbers, have something to do with ostension. A nice example can be found in the book by Freudenthal [1960], where the design of lessons of a cosmic language to be addressed to another civilization starts from ostensive definitions of names of some numbers (objects to be displayed are radio signals grouped in one-element sets, two-element sets, etc.).

also in mathematics, and every ostensive definition, as well as every other definition consists in the coordinating of an expression with a thing, be it a physical object, an abstract object, or still something else. The presence of a thing in the process of defining is most conspicuous in the case of ostensive definitions, and in this sense the ‘deviant’ case of ostensive definition, paradoxically, can be regarded as a paradigm of definition in general (cf. Subsec. 3.1 below). Therefore it is advisable to discuss ostensive definition at the outset, even if some orthodox logicians should perceive this as turning things upside down.

Ostension is closely related to the instinct for generalization which was discussed earlier (Seven, Subsec. 5.1). To see this connection (to be discussed below in Subsec. 1.2), let us first consider an example of ostension.

Suppose that one is entrusted with the task of giving a name to a thing which is shown to him. This is not a typical case; a typical **ostension** takes place when a person who knows an expression teaches another person its meaning (in the language in question) by hinting at a suitable specimen of the relevant kind and by uttering the statement ‘the name of this object is such-and-such’. However, this non-typical case in which only one person is involved is more convenient to start with. Such a procedure is nicely reported in the following passage of the Bible.

The Lord God formed every beast of the field, and every fowl of the air; and brought them unto Adam to see what he would call them; and whatever Adam called every living creature, that was the name thereof. (Gen. 2, 19)

This definitional situation consists of the following elements. There is a thing which appears to us, a name we introduce to call that thing, and a performative sentence to express the creative FIAT: let the thing be called ‘such-and-such’. The phrase encircled here with commas is a variable for a metalinguistic name, i.e., the name of that name which we give the thing. Thus, there was presumably a moment in which Adam said: let this be called ‘tiger’.

The ‘let’ form is to express the act of terminological creation. It has to be replaced by another form when the creation is accomplished, and one wishes to communicate its results to someone

else. This may have been the situation of Adam and Eve (created after Adam's definitional activities). Having once called an animal a tiger, Adam may have communicated his definition to Eve with words like those:

(OQ) The name of this object is 'tiger'.

Or, simply

(OU) This is a tiger.

The existence of two such forms, referred to as OQ (for Ostensive definition which Quotes a term) and OU (for Ostensive definition which Uses a term) should not induce us to think that there are two kinds of definition corresponding to these forms, one dealing with the expression 'tiger', the other with the tiger itself. Obviously, the form OU has two functions, and in the other function it expresses only the recognition an object as a tiger without any definitional purposes. It is a context, or a special intonation (which Adam might have used when teaching Eve the language he invented) or a typographical device (as the abbreviation 'df' for 'definition' preceding the statement in question) which indicates the definitional role in which a statement of the form OU is equivalent to a statement of the form OQ.

It is a reasonable advice, both logical and rhetorical, that one should preferably use the form OQ rather than OU to avoid misunderstandings, unless the above-listed means (to hint at the definitional function of the uttered statement) are sufficiently readable for the audience. On the other hand, the form OU should be admitted as a more economical stylistic variant for the convenience of people suitably skilled in linguistic expression and interpretation.

1.2. In the ostensive defining there manifests itself the instinct for generalization (cf. Seven, Subsec. 5.1), i.e., a tendency which, like other instincts, is rather a biological drive than a cultural norm; it often works independently of the will of the subject in question, even if the subject is not aware of its functioning. One cannot help generalizing, this propensity is not only part of the animal instinct for survival but also a creative cognitive force in humans.

True, there occur frequent and serious errors in generalizing. However, this fact should not be dramatized, for — as a rule — each generalization is a hypothesis to be tested and, possibly, to

yield to another hypothetical generalization which in the given moment would better stand up to its test. Contrary to the opinion of Aristotle and even of the 19th century philosophers of science, human cognition is a risky enterprise in which certainty is the ideal extreme to be sought but not to be enjoyed in the actual world (in this perspective logic and mathematics are closest to certainty and thus constitute the actual extreme). On the other hand, this situation is not so uncomfortable as it may seem because generalizations should be relativized to questions posed at a given time, and may match these questions, even if they are not able to stand up to new ones. Thus Newton's mechanics was perfectly sufficient with regard to the questions and needs of science and technology for two centuries, and in this limited range it continues to be valid; it is the range of, so to speak, the medium-sized world, i.e., neither that of the macrocosmic (cosmological) nor that of the microcosmic (subatomic) scale. As we shall see, the merits and flaws of our everyday prescientific generalizations (to be dealt with by cognitive rhetoric) should be subjected to analogous relativization: a generalization which is wrong for some purposes may prove adequate for some other testing questions or purposes, and in this sense it may enjoy a limited certainty (it is why our reason, despite being so fallible, may succeed in handling matters of everyday life).

The nexus between generalization and definition can conveniently be shown in the case of ostensive definitions. When Adam in Paradise gave animals their names, in each case he must have decided *why*, i.e., in view of which properties, should the given name belong to the animal in question. Should a creature be called 'tiger' for its having four legs alone, or for having also such and such size, colour, shape, and habits? The more properties are included into the content of the term being defined, the greater the possibility of falsifying the generalization, since grouping them under one term is equivalent to the statement that each of them implies each other. Suppose that the creature in question brought to Adam by God was a male, and physical features of a male were included by Adam in the content of the term 'tiger'; a biological theory involving the statement that all tigers (as characterized by the earlier listed properties) are males would be totally unable to account for the procreation of tigers, and so it would fail to stand

up to a crucial test, but for some limited purposes of practical life in Paradise it might work.

The procedure of ostension shows up still other features of any definition, namely the fact that definitions must be introduced on the ground of a theory. This statement may seem unsettling to those who share the empiristic doctrine, especially in its neopositivistic version, that there is a basis of empirical concepts which are absolutely primitive, and this basis consists of expressions having been introduced to the language in the ostensive way. Should one succeed in demonstrating that even ostensive procedures must presuppose some theoretical assumptions, then *a fortiori* this result would hold for more advanced definitions.²

The above-mentioned language for cosmic intercourse, devised by Freudenthal [1960], provides us with an instructive example of conceptual presuppositions necessary for efficient ostensive procedures. If a cosmic receiver of the set of three radio signals accompanied by a conventional signal to denote the number three (as exemplified by the three-signal set) is to correctly interpret the message, he must have previously had the idea of communication, especially the idea of defining, then the idea of ostensive defining, and the idea of number; the last is specific to the reported case, the other ones belong to every ostensive procedure. The ideas necessary in Adam's case were those of colour, shape, size, etc., because naming something tiger for having yellowish fur with black bands presupposes the awareness that yellow and black are colours. In this context I preferred the term 'idea' over 'concept', following the choice of Leibniz who reserved *conceptus* for better defined and more precise thoughts and *idea* for those remaining vague, even not fully verbalized and realized, but nevertheless efficiently controlling cognitive processes.

If an empiricist helplessly asks where such preconceptions come from, he should blame himself for denying any kind of innate knowledge or inborn cognitive skills. That there is a knowledge or skill recorded in animal cells is no supernatural mystery to be renounced by modern enlightened minds. And these **internal records**

² This problem is treated more extensively by Kotarbińska [1960] and by Marciszewski [1966].

are pre-theoretical seeds of later consciously and verbally developed theories (following Stoics and Augustine, we might call them *rationes seminales*, i.e., a kind of cosmic software). Since there are enormously many degrees of conscious realization and verbalization, for brevity I subsume those pre-theoretical ideas under the concept of theory, though the deeper we go into the biological (as opposed to cultural) layers, the less reason there is to speak of concepts or theories.

The next comment, necessary to explain the set of notions proposed for handling definitions, is concerned with the relation between concepts and theories. Again, this relation is considered against the background of the above mentioned Leibnizian *lex continui* enabling one to see connections which would otherwise have remained unnoticed. Let it be noted that if there is a set of primitive concepts, i.e., such that none of them is defined in terms of earlier introduced ones, then these concepts, in order to be understandable, have to be interconnected with one another in a way which contributes to explaining the content of each of them. Such interconnections must be expressed in the form of some propositions (even if un verbalized), as can best be seen in sets of axioms of deductive theories. Obviously, such a set of propositions must have consequences, and those together with axioms form a theory. Owing to the said *lex continui* we can consider various degrees of awareness associated with such concepts, degrees of verbalization and precision, and the deeper we go 'back' (from our conscious cultural level) the less there is actuality and the more mere potentiality, but we should not fail to investigate that 'underground world' if we intend to reach the best possible understanding of the phenomena of cognition and communication.

2. Normal definitions of predicates and names

2.1. The term **normal definition** stands for a statement which makes it possible to eliminate a newly introduced expression by replacing it with older ones. A normal definition is a genuine definition in the traditional sense, hence when speaking about a definition without any adjective we mean a normal definition.

Normal definitions have the form of equivalences or identities (as discussed in Chapter Six); the symbol of equivalence or of

identity, sometimes accompanied by the abbreviation ‘df’, is said to be a **definitional functor**. The expression being defined, say A , which occurs on the left side of the definitional functor is called the **definiendum**, and the expression used for defining what occurs on the right side is called the **definiens**. The form of a definition depends on the syntactic category of the expression to be defined.³ Names and name-forming functors can be defined by identities, while definitions of sentence-forming functors (such as connectives, predicates, etc.) must take on the form of an equivalence. Let the following formulas exemplify the typical forms of definitional equalities and equivalencies.

Of a name-forming functor: ‘the successor of x : $S(x) = x + 1$.

Of a name: $1 = S(0)$.

Of a name-forming functor: $x - y = z \equiv z + y = x$.

Of a sentential connective: $(p \rightarrow q) \equiv (\neg p \vee q)$.

Of a predicate: x is even iff x is divisible by 2.

In these examples definitions are formulated without any formal index distinguishing them from theorems. Such an index is not necessary when its function is indicated by the context. Otherwise we write ‘df’ either as the subscript (or the superscript) of the definitional functor or as the prefix of the whole definitional formula (the latter is advisable when combined with a numbering of successive definitions), for instance:

$S(x) =_{df} x + 1$,

Df.1 $S(x) =_{df} x + 1$.

In a natural language we have at our disposal some convenient stylistic variants of definitional forms, sometimes assisted by such typographical devices as italicizing the definiendum, for example:

(a) a number is said to be *even* if it is divisible by two;

(b) ‘even’ denotes a number divisible by two;

(c) ‘even’ means the same as ‘divisible by two’.

In the first of these examples the context of the construction ‘... is said to be ... if ...’

is to indicate that we deal with an equivalence which otherwise would be stated in a more clumsy form using the phrase ‘if and only if’ or, for brevity, ‘iff’. One may also use the form:

³ The notion of syntactic category is sketched in Chapter Six, Subsec. 1.1.

(d) a number is *even* iff it is divisible by two.

The last form, that is one which does not use either quotations marks or devices like the phrase ‘it is said’, provides an opportunity for pointing to the distinction between **object language**, i.e., a language dealing with extralinguistic entities, and **metalanguage**, i.e., a language describing another language. A typical device to form a metalanguage is **quotations marks** in the form of, e.g., **inverted commas**. With the help of inverted commas we form names of expressions, for instance the first word in example (b) is a **metalinguistic name** of an expression which refers to even numbers (as mathematical entities). Also the phrase ‘is said to be’ as used in example (a) concerning an act of speech belongs to the metalanguage of English.

It is held by some logicians (e.g., Kotarbiński [1929]) that a genuine normal definition is a **nominal definition**, i.e., a terminological statement about the meaning of an expression; in other words, that it is a metalinguistic statement. This claim evidently disagrees with some practices of mathematicians. A typical form to be found in mathematical papers, textbooks, etc., can be exemplified by the following definition: *A right angle is any one of the four angles between two lines that intersect so that adjacent angles between them are equal.* (*Mathematics* [1975], p. 150). There occurs, however, a corresponding metalinguistic form; e.g., in Euclid himself the same definition of a right angle is worded with the phrase ‘is called a right angle’ which follows the description of properties of the object in question (Book One, definition 10).

Why are mathematicians, who should be most sensitive to the preciseness of the language they use, so careless that they ignore the distinction between an object language and a metalanguage? Should their habit be imitated in rhetorical practice, or rather the latter should prove more cautious? It is the latter option which is advisable, and the reason is that whenever a mathematician defines the name of an object he can be sure that the object for which the name stands for does exist; then the metalinguistic form and the object-linguistic form are equivalent; this issue will be discussed later, in the context of correctness conditions of the definitions of names. First, however, we should consider the rules of defining predicates.

2.2. To be correct, a **definition of an n-place predicate** P should take the form

$$P(x_1, \dots, x_n) \equiv A$$

and satisfy the following conditions:

(1) x_1, \dots, x_n are distinct variables, that is, every variable may occur only once in the definiendum.

(2) No free variables other than x_1, \dots, x_n occur in the definiens A , that is, every variable which occurs free in the definiens should also occur free in the definiendum.

(3) In the definiens, the non-logical constants should be either primitive or previously defined in the theory.

The following comments may explain the meaning of these conditions.

Condition 1 would be violated if, for instance, the binary predicate of the set-theoretical inclusion were defined as follows:

$$(X \subset X) \equiv X \cap X = X.$$

The obvious failure arises in that we are actually defining the one-place predicate ‘included in itself’; hence the symbol ‘ \subset ’ could not be eliminated from contexts like ‘ $N \subset R$ ’ (read N – natural numbers, R – real numbers). The need of applying this condition to definitions in a natural language is hardly felt since natural languages do not have devices analogous to longer sequences of variables (even if there are some counterparts of single variables).

Condition 2 would be violated by the following attempt to define the synonym of sentences:

Df(syn): A is said to be *synonymous with* B if there are such inference rules in a language L , that A is derivable from B and B is derivable from A .

In this attempted definition Df(syn) the variable L occurs free in the definiens and does not occur in the definiendum at all, contrary to condition 2. Let A_0 and B_0 be the sentences substituted for A and B , respectively, and let L_1 and L_2 be the names of languages substitutable for L occurring in the definiens (alone). Suppose that A_0 and B_0 are derivable from each other in L_1 , but not in L_2 . However, this lack of derivability in one of these cases contradicts the consequence of Df(syn) to the effect that derivability has to hold for any case whatever, according to the law of binding a free

variable in the consequent of a conditional, provided it is not free (i.e., it either does not occur at all or occurs as bound) in the antecedent. This law runs as follows: $(C \equiv D(x)) \rightarrow (C \rightarrow \forall_x D(x))$, where x does not occur in C as a free variable.

Condition 2 can be applied to some structures of a natural language. The error illustrated by Df(syn) can be reproduced in English as follows: “Two expressions are said to be *synonymous* if they are derivable from each other in a language”. This obviously leads to such inconsistencies as the one following from the formula Df(syn); for instance, there is a language TE, viz., a Theological version of English, in which the expressions ‘something is a religious truth known to people but unattainable by human reason’ and ‘something is a truth made known by God’ are derivable from each other; by virtue of the above counterpart of def(syn) it follows that these sentences are synonymous in an absolute sense (i.e., not relativized to a language, say TE) which is a wrong conclusion. This error arises from using the phrase ‘a language’, being a counterpart of L as a free variable, in the definiens. It will be rectified if one introduces the same variable into the definiendum and prefixes the whole definition by the universal quantifier to bind that variable in such a way as this: for any language, two expressions are said to be *synonymous* in it if they are derivable from each other in it. Here the pronoun ‘it’ plays the role of a variable ranging over languages, while the universal quantifier ‘any’ ensures that in both occurrences ‘it’ refers to the same language.

As for Condition 3, a typical infringement of what it requires is called the **vicious circle in defining**. Let this fallacy be compared with the **vicious circle in reasoning** which consists in using the statement to be proved in the role of a premise; e.g., the view that Leonardo’s paintings are masterpieces is deduced by someone from the view that Leonardo was a man of genius, and when he is asked to substantiate this view, then he mentions the fact that Leonardo created artistic masterpieces. The same example, suitably reformulated, can illustrate the definitional vicious circle: let the expression ‘a man of artistic genius’ be defined by the expression ‘a man who is able to create masterpieces’, and then ‘masterpiece’ be defined as ‘a product of artistic genius’. In spite of being so obviously erroneous, such slips are frequent in everyday arguments, certainly because of deficiencies of human attention and memory.

However, we should not be over hasty in blaming such a manner of defining; sometimes it may have a rational, even if unconscious, justification overlooked by logical pedants. Namely, concepts of natural language are subjected to frequent modifications caused by contexts of their use, therefore what resembles the fallacy of vicious circle may reasonably contribute to old meanings by completing it with new contexts. Such concepts as ‘man of genius’ and ‘masterpiece’ form two contexts (even if created on the pattern of vicious circle) which shed light upon each other as far as their meaning and extension are concerned. The meaning of the term, e.g., ‘man of genius’ is far from being precise; it is vaguely felt rather than duly defined, hence a hint at the relation of its meaning to the meaning of the term ‘masterpiece’ may contribute to making each of them less vague. Such a defence of circularity should not be adopted in the case of circular reasoning, but the case of defining is different because of the mentioned interaction between meanings. This problem will be discussed further later on, with reference to the theory of axiomatic definitions. Now, continuing the survey of normal definitions we should deal with definitions of names and functors.

2.3. In the language of modern logic unlike in natural languages there is a sharp demarcation line between predicates and names, which is also reflected in the theory of definition.⁴ Since the term ‘name’ is needed both to cover individual names in predicate logic and general names functioning in natural languages, it is advisable in the present context to resort to the term ‘individual constant’ as a more precise counterpart of the term ‘name’. An expression is said to be an **individual constant** if it denotes an individual as does, e.g., a proper name or (in a suitable context) a personal pronoun.

To be correct, a **definition of an individual constant** c in the form $c = y \equiv A$ must satisfy conditions 2 and 3 (as above); condition 1 is disregarded since in this form there cannot occur more

⁴ In natural language this difference is blurred by the fact (discussed in Chapter Six) that the same lexical item may function in different syntactic roles, especially as a grammatical subject, hence as a name, and as a predicate. On account of this fact, the logical theory of definitions should be adjusted to natural languages.

than one variable. Furthermore, such a definition should satisfy the **condition of definiteness** combining the **condition of existence** with the **condition of uniqueness**.

The term ‘condition of definiteness’ is not usual in logical texts since existence and uniqueness are never separated in them. When applying the logical theory of definitions to natural languages, we are sometimes bound to consider them separately, hence for the cases in which they are inseparable it is convenient to have a special term; for that role I suggest ‘condition of definiteness’ since the *definite* descriptions are those terms which always satisfy both conditions.

A theory affected by a definition infringing upon the definiteness condition has to incur contradiction. If the existence condition is violated, that is, no entity is referred to by an individual constant, then contradiction results from the law of existential generalization:

$$\varphi(a) \rightarrow \exists x \varphi(x);$$

the corresponding inference rule is that of *Introducing the Existential Quantifier*. Obviously, if ‘Pegasus’ is used as an individual constant in the sentence, e.g., ‘Pegasus is a wise horse’, then it follows that there is an individual which is a wise horse.

If the uniqueness condition is infringed, then there are at least two different individuals named with the same individual constant, say a and b , both named ‘ c ’. This means that $c = a$ and $c = b$, hence $a = b$ which denies the assumption of their difference (that is, $a \neq b$) and thus makes the theory inconsistent.

2.4. Why is the definiteness condition obligatory for individual names and is not for predicates? What about general names which are ignored by modern predicate logic, but not ignored by traditional logic nor by natural languages? The answer to the first question should help to answer the latter.

Predicate logic is closely tied with set theory, which provides logic with the entities which its symbols stand for. There is an astonishing simplicity in the set-theoretical ontology. Two categories of objects correspond to an infinite variety of syntactic constructions, namely individuals and sets (which in some contexts I prefer

to call classes for some useful associations). This miracle is possible thanks to the fact that sets themselves can form an infinite hierarchy and be ordered in various ways.

Against this background certain predicates are seen as expressions denoting sets of some sets. For instance, some arithmetical predicates, such as those concerning addition, multiplication, etc., are construed as denoting sets of ordered triples (i.e., three-element sets) of numbers; thus the predicate ' $Sum(z, x, y)$ ', to be read ' z is the sum of x and y ', refers to the infinite set of triples of, say, natural numbers, as $(0,0,0)$, $(1,0,1)$, $(1,1,0)$, $(2,1,1)$, $(2,0,2)$ and so on, in infinity. This description is concerned with predicates referring to relations since relations are defined in set theory as ordered sets. As for non-relational, that is, one-argument (one-place, unary) predicates, they refer to sets of individuals, e.g., the predicate 'is an odd number' is predicated of arithmetical individuals being odd numbers.

In this conceptual setting, predicates are never deprived of reference; each of them enters the language of an applied predicate calculus as coordinated with a set, which may, in particular, be the empty set. There is no such privilege granted to individual constants, and therefore it is necessary for each such constant to prove its non-emptiness, i.e., that it has a reference. As for the condition of uniqueness, an analogous condition should be stated for predicates (though logic textbooks fail to state it, presumably because of its obviousness); the infringement of this condition could be called **double-denotation fallacy**, that is the error of attaching two different sets (as denotations) to one predicate expression (the same designation could be applied to the case of individual constants which fail to satisfy the uniqueness condition). Also this error leads to a contradiction. Suppose that (i) there are two different sets A and B , that is to say $A \neq B$, and (ii) they are assigned to the same predicate ' P '; then it follows (from ii) that:

$\forall_x(P(x) \equiv (x \in A))$, and

$\forall_x(P(x) \equiv (x \in B))$; hence

$\forall_x((x \in A \equiv (x \in B)))$, i.e.,

$A = B$, contrary to the assumption (i).

Natural-language arguments can hardly be expressed in predicate logic. Instead of predicates we use general names, and also

individual constants can be replaced by suitably transformed (e.g., by articles) general names. This discrepancy between the modern logical orthodoxy and the logic practised since time immemorial in natural languages (roughly approximated by Aristotelian syllogistic) is a thought-provoking phenomenon. It may be due to deep differences between a spoken and a written language; it is only in the latter that one may take full advantage of the technique of variables whose application range is being suitably restricted by predicates, while in natural languages the range in question is being indicated by a general name (this is just a conjecture to be developed and tested in an opportune context).

Though the semantics of general names seems to exceed that substantiated by set-theoretical ontology, the latter can be expected to provide us with some data useful for the semantics of general names, and for corresponding rules of definition to deal with existence and uniqueness. We shall restrict our attention to what may be called genuine or proper general names in contradistinction to apparent ones, analogously to the distinction between genuine individual constants and apparent ones as, e.g., 'Pegasus'; they are alike with respect to some linguistic (syntactic) rules of use, but not with respect to certain logical (semantic) rules. I shall employ the traditional term **universal** (as a noun, not an adjective) to distinguish the references of genuine general names.

Let the (supposed, as yet) reference of a general name be termed a 'universal'. Let a set be said to be a **universal** if it meets the following requirements: (i) it is non-empty; (ii) it is potentially infinite, that is, it is not defined by the enumeration of its elements but by a property common to all elements, so that any new object possessing that property is counted as a member of the set; (iii) it is embedded in a relational structure of sets being the domain of a theory endowed with a desired explanatory power.

The above list of requirements is adjusted to some common practices and intuitions. Universals are traditionally meant as being related just to essential properties, such as that of being a man, and not accidental ones, such as being an inhabitant of Berlin (which does not satisfy either (ii) or (iii)). Obviously, condition (iii) is burdened with a vagueness which in many cases makes it impossible to decide whether we have to do with a universal or

not, but for the present purposes it suffices that at least in some cases the question can be settled.

Thus defined, the concept of universals hints at a possibility of stating the condition of definiteness for general names. This requirement is met if there is a universal referred to by the name and, at the same time, it is unique; uniqueness, as in the case of predicates, consists in attaching exactly one universal to the name in question, i.e., in avoiding the double-denotation fallacy.

Owing to the introduction of the concept of universal to provide general names with a reference, the new item should be added to the list of forms of normal definitions as given at the outset of this Section. As yet, we have no formal means to present the structure of the definition of a general name because of the lack of appropriate means in predicate logic, in which the syntactic category of general names, as construed above, does not occur. The closest neighbour is the category of names made from predicates with the help of Hilbert ϵ -operator, in whose definition there explicitly occurs item (i) from among the three above-mentioned conditions.⁵ Unfortunately, the language of predicate logic does not suffice to express the two other conditions. In particular item (iii) as resorting to some concepts of methodology of sciences cannot be handled solely in terms of predicate logic. Its significance from the rhetorical point of view has to be shown in connection with a philosophical doctrine about mind, language and logic which is outlined in next sections.

3. The holistic doctrine of definition

3.1. The traditional theory of definition, going back to Aristotle, claims that each definition characterizes a species of things by mentioning its genus (kind) and what distinguishes the species in question from the other species of the same genus. This was stated in the famous maxim *definitio fit per genus et differentiam speci-*

⁵ This operator forms a name to be used in the member supposition, i.e. referring to an arbitrary element of the class denoted by a predicate, provided that the predicate in question is not empty. See Hilbert and Bernays [1934-39], Beth [1959], Bell [1993].

ficam. Genus and species referred to in this context belong to the category of universals discussed in the previous section.

At the same time, it was hardly possible to refuse the label of definition to some other kinds of statements, e.g., terminological explanations as found in dictionaries, hence the characterization of a species of things through a kind and a specific difference has been called **classical definition** to distinguish it from other types of definition. It held a distinguished position in the structure of scientific or philosophical theories, while other kinds played only an auxiliary role. In this sense the traditional theory can be termed a **monistic doctrine of definition**.

On the other hand, with the decline of Aristotelian metaphysics, upon which this monistic doctrine was based, there appeared another kind of monism. This new doctrine grew so dominant that it took over even the label of ‘traditional conception’ from the old Aristotelian doctrine. According to this new tradition, the only function of a definition consists in its being a convention with regard to the use of a language. Such a convention results in introducing a new expression or combination of symbols, called *definiendum*, to replace another combination of expressions, called *definiens*, whose meaning is already known in terms of certain already existing expressions. Thus definitions are considered as a kind of shorthand which can be, theoretically at least, dispensed with altogether; it has no relation to the content of the theory considered, but only to the language in which it is expressed. However, this second monism proved as untenable as was the first. Not only the fact that the definiteness condition ties a definition to an extra-linguistic reality, but even more the existence of recursive definition in mathematics (see Section 4 below) compelled theorists of definition to acknowledge its kinship with some statements about things.⁶

Such being the case, it was tempting to adopt a pluralistic position which admitted of more than one kind of definition with respect to the question of what definitions refer to, whether to things or to expressions of a language. That position was typically represented by Ajdukiewicz [1958] and [1974], in the latter expressed

⁶ See, e.g., Curry [1958]; the above characterization of the second monistic doctrine is taken from the beginning of that paper.

even in the title ('Three concepts of definition'). The term **nominal definition** was reserved for definitions of expressions, and the term **real definition** for definitions of things. This compromise, though, creates a new problem, also handled by Ajdukiewicz, namely the question of what should be regarded as common to real and nominal definition: finally, the only feature shared by them is that each of them is a univocal characterization of something, the former of things, the latter of expressions; this is too little, indeed, to deserve a common denomination.

To disentangle us from these problems, let us try to see the role and the corresponding structure of definition in a new way. This new approach starts from asking about the structure of the predicate 'is definition'. It is usually taken for granted that it should be a binary, i.e., two-argument, predicate of the structure ' $D(x, y)$ ', where ' D ' means 'is the definition of', ' x ' represents the statement being the definition, and ' y ' represents something being defined, either an expression or a thing. Should we stick to that point as an irrevocable dogma?

Once having agreed that the procedure of ostension should be a paradigm for any definition altogether, one is prepared to describe acts of defining by the following six-argument predicate

$$D(x, e, L, c, t, r),$$

that is to say: x is the definition of the expression e_L (definiendum in Language L) by characterization c intended to introduce the thing t into the domain of L , and to produce the internal record r of t in the mind-body of an addressee.

An **internal record** (cf. above, Subsection 1.2) is conceived as belonging both to one's body, where data must be somehow physically stored (as in computer memory), and to one's mind by which the data can be read out in appropriate conditions (e.g., in an act of remembering). The internal record is produced by what is above called characterization, and comprises such various stimuli as verbal descriptions, drawings, models, etc.; in an ostensive procedure it can be said that we deal with a model, namely a physical instantiation of a general object (i.e., a universal).

This approach takes into account all the elements involved in the process of defining in their mutual relations forming a whole,

it can thus be termed the **holistic doctrine of definition**, opposed to both monism and pluralism. Its holistic nature consists also in taking into account all the three kinds of semiotic relations, viz., syntactic, semantic and pragmatic connections. A definition *defines* an expression (syntactic aspect), *introduces* a thing (semantic aspect), and *produces* an internal record (pragmatic aspect).

According to the holistic approach, a nominal definition (cf. 2.1 above) should rather be called an **asemantic definition** to point at the lack or irrelevance of the semantic component. The term ‘real definition’ proves then unnecessary as any complete definition is concerned with a thing belonging to extra-linguistic reality.

In certain situations some members of this whole prove irrelevant to the purpose in question and are then disregarded. For instance, when we propose a new term to abbreviate a longer phrase (e.g., ‘radius of a circle’ for ‘straight line drawn from the centre to the circumference’), when the existence of the thing denoted by the latter is granted (as is granted the existence of a circle’s radius owing to postulate 3 in Book One of *Elements*), then the lack of a mention about the thing itself does not do any harm to our theorizing. In other cases, the explicit mention of the thing in question must be made in the form of the proof of its existence.

All the six factors occur in the ostensive definition. What is remarkable about it is that the characterization c , which is usually provided by a verbal description, in the case of ostension is provided by a physical object bearing a special relation to the introduced thing t ; this relation consists in instantiating the thing by the physical object shown in the role of characterization. The occurrence of this relation helps us to realize how the thing t is referred to by the expression e . For instance, when ostensively defining the thing ‘yellow’ (being, presumably, a universal) one produces, e.g., a piece of lemon and utters the phrase ‘this is yellow’ so that the lemon provides the intended stimulus on the retina and somewhere deeper in the brain so producing an internal record to represent the thing.

Whether such stimulating characterization is provided by a physical object or by a verbal description depends on the stage of construction of the language in question. To begin with, we absolutely need ostension, but later on a verbal description can do

as well. After the notion of lemon and that of colour are introduced to the language, the procedure of ostension can be replaced by a purely verbal explanation, as ‘yellow is a colour like that of a lemon’. Thus the ostensive definition is not deprived of anything that essentially belongs to the act of defining, while the fact that the thing introduced is somehow instantiated in this act helps us to realize that basic truth about definitions that not only a piece of language but also a piece of reality is what we deal with in defining.

3.2. The lesson that definitions deal also with things, and not with language alone, is necessary to understand the role of definition in theorizing, especially in developing scientific theories. Were every definition just a verbal move which makes our texts shorter but does not advance our knowledge about the world, the strategies of cognition and of communication would be something very different from what they actually are.⁷

This claim may seem to oppose the commonly acknowledged postulate that every normal definition should meet the criterion of **eliminability**, to the effect that it be possible to replace any statement containing a defined expression by an equivalent statement not containing that expression. In orthodox expositions of the logical theory of definition, this criterion is accompanied by that of **non-creativity** which states that a definition should not function as an axiom, that is, whatever is provable in a theory on the basis of the axioms with the definition added to them has also to be provable without that definition.⁸

To avoid misunderstandings it should be stressed, first, that these criteria have been stated to hold for deductive formalized theories, and no one has dared to claim their validity for empirical

⁷ Cf. requirement (iii), of Subsection 2.4 above, that the definition of a thing named with a general name should increase the explanatory power of a theory. For mere shorthand we need not such a serious term as ‘definition’; the term ‘abbreviation’ would do, as the situation is not much different from that in which the abbreviation ‘AI’ replaces the term ‘Artificial Intelligence’.

⁸ See, e.g., Suppes [1957], Ślupecki and Borkowski [1967], Grzegorzczak [1974], *Logic* [1981]. These criteria are emphasized by Polish authors, in accordance with the teachings of S. Leśniewski; in the Anglo-Saxon world they became wider known due to P. Suppes who explicitly quotes Leśniewski.

theories; second, even in regard to formalized theories it should be remembered that Leśniewski had his own rigorous doctrine of formalization which rather postulated a reform of mathematical practice than tried to do justice to it. This practice, however, does not obey the criterion of non-creativity, as duly observed by Curry [1958] (in spite of his formalistic tendencies), in one case at least, namely in the case of a **recursive definition**, called also **inductive definition** or definition by induction.

The creative function of definitional recursion is conspicuous in its introducing a new object which otherwise would not appear altogether in the domain in question. Such creation is performed in two steps. In the initial step it is unconditionally specified which objects belong to a given set, e.g., that 0 belongs to the set of natural numbers; in the induction step it is specified which objects belong to the set in question provided that the objects listed in the initial condition belong to it. As an example take the definition of addition (S is the successor function).

$$\begin{array}{ll} y + 0 = y & \text{the initial step} \\ y + S(x) = S(y + x) & \text{the induction step} \end{array}$$

The object introduced by the above definition belongs to the category of **functions**, or **operations**, which together with natural numbers, as individuals, inhabit the domain of **Peano arithmetic** (so called after Giuseppe Peano, 1858–1932, who first axiomatized the arithmetic of natural numbers). Functions are genuine mathematical entities which are ordered sets, e.g., the function of addition is identical with the set of ordered pairs resulting (successively, so to speak) from 0 and 0, 0 and 1, 1 and 0, 1 and 1, and so on into infinity. Without the above inductive definition, no such object would enter the domain of arithmetic. One may ask whether the listed pair of equations constitutes a normal definition; the answer is as follows: the literal form of normal definition can be trivially obtained when the equations are preceded by a clause like that: “The function $\varphi(x, y)$ is that of *addition* if and only if it satisfies the following conditions:” (here follow the equations).

In empirical theories, that is to say, those which extend our knowledge beyond the limits of sensory experience, ever new objects are being introduced. Being new they require naming, and that act of introducing an object blended with the act of naming is

what is being done by definitions. Of course, one can introduce as many objects as one wishes (there are many trivial ways of doing so, e.g., to make pairs, triples, etc., of already existing objects), and one can define ever new names for objects so introduced. However, the introduction of new things should have a cognitive purpose, especially it should increase the explanatory power of our theories, otherwise it would amount to a praxeological fallacy. Thus the rationale of condition (iii), as stated in Subsection 2.4 above, can now be seen more clearly.

In order to scrutinize the creative function of definitions in empirical theories, the function expected to throw light on the nature of human **intelligence**, we need a preparatory study concerned with an analogous but considerably simplified situation. Such a situation can be found in deductive axiomatized theories due to the fact that their axioms introduce the objects which the theory is to handle. As shown above, new objects can be added by inductive definitions, thus the question arises of whether there are still other definitional methods of introducing new objects in a theory. Axiomatized theories, owing to their simple and conspicuous structure, provide us with excellent material to begin with.

4. Implicit definitions and conclusive conceptualization

4.1. The concept of **axiom** is as old as European logic and mathematics themselves. In its Greek form $\alpha\xi\omega\mu\alpha$ it occurs in both Aristotle and Euclid, though in the latter the proper axioms are listed under the title of **postulates**. Nowadays these two terms are regarded as similar in meaning, though not to the extent of stylistic interchangeability (e.g., only the former gave rise to the adjective ‘axiomatic’ and the noun ‘axiomatization’). Definitions were also considered as essential elements of deductive systems, by both Aristotle and Euclid, as well as their followers, but an essential similarity between their function and the role of axioms was not recognized until the studies of definition brilliantly carried out by the French mathematician Joseph Diez Gergonne (1771–1859), since 1832 distinguished as a member of the Prussian Academy of Sciences in Berlin (cf. Kamiński [1958]).

Gergonne [1818] suggested the term **implicit definitions** (*définitions implicites*) in contradistinction to what he called **explicit**

definitions (*définitions explicites*); the latter can roughly be identified with what is now called normal definitions. According to Gergonne, an implicit definition of a word is a context in which this word is involved so that the way of using it in the context makes it possible to correctly guess its meaning.

There was a long route from that vague intuition of Gergonne to the modern methodological investigations on the definability of concepts as conducted esp. by Tarski [1935] and Beth [1953]. In clearing up the relationship between axioms and definitions an important role was played by the study of independence of axioms. This is a glorious story, concerning both mathematics and logic, which begins with attempts to prove the independence of Euclid's parallel axiom (e.g., G. Saccheri, 1667-1733) and leading to the discovery of non-Euclidean geometries. The methodological awareness of the significance of these mathematical results is due to the founders of modern logical systems, as Russell, Frege, Hilbert, and Warsaw School logicians (esp. Łukasiewicz and Tarski). It has become the standard that for each logical system its author is bound to prove that no axiom can be inferred from the remaining ones (an efficient method of doing so is due to many-valued logics). Parallel to this problem is that of the independence of concepts appearing in the axioms: it should be proved that each concept used in the axioms of a given system is actually primitive, that is to say it is not definable in terms of the remaining concepts found in the axioms.

Thus the function of axioms as the first premisses and their definitional function have become clearly distinguished. This, in turn, paved the way for the realization of how axioms in their own way, different from that specific to normal (explicit) definitions, do contribute to the interpretation of primitive terms occurring in axioms.

According to the standard methodological theory, axioms introduce some objects, those referred to by primitive terms, and at the same time they characterize their meaning in the way typical of a context of use. According to the same theory, definitions do not introduce new objects, they merely rename the old objects for reasons of technical convenience. The case of inductive definitions, as

reported below, blurs that clear picture, and obliges the methodology of mathematics to look for a more sophisticated solution (as, e.g., the attempt by Curry [1958]). However, it is not the aim of the present discussion to deal with the problem of definitions in mathematics at such an advanced level. Instead, we should learn from the mathematical practice that elementary truth about the definitional role of axioms, and then make use of this lesson as far the definitional introducing of new objects to empirical theories is concerned. Before we engage in such discussion, some examples of how axioms play their definitional role will be in order.

4.2. Instructive examples can be found in a theory being in a way fundamental for modern mathematics, and at the same time being an excellent object for methodological studies. I mean arithmetic as axiomatized by G. Peano.

Here are the axioms of Peano's arithmetic of natural numbers.

1. 0 is a number.
2. Every number n has precisely one successor $S(n)$.
3. 0 is not the successor of any number.
4. Distinct numbers have distinct successors.
5. If a set of natural numbers contains the number 0 and contains, together with any number, also its successor, then it is the set of all natural numbers.

Axiom 1 introduces the object 0, and so it states about the domain in question \mathbf{N} (for Natural numbers) that it is not empty; it is the first step towards defining this domain. Axiom 2 introduces the new object S (for Successor) which is presented as a one-argument function. Axiom 3 combines information of the preceding axioms so that we learn something new both about 0 and about S : that 0 cannot be the value of S , it can occur just as the argument; hence, if there is any ordering of \mathbf{N} due to S , in the sense of the successive introduction of the values of S , then 0 should occur at the very beginning of the sequence obtained.

That there is such an ordering we learn from the next axiom. According to 4, it cannot be so that, e.g., $S(0)$ and $S(S(0))$ are both successors of 0, since 0 and $S(0)$ are distinct numbers; hence there results the following ordering: 0, $S(0)$, $S(S(0))$, etc. Axiom 5 is of special import as it gives the justification for the method

of mathematical induction. Its most elementary consequence is to the effect that the successor function when applied successively starting from 0 generates the whole set of natural numbers, and that is to mean that this set, i.e., the domain of arithmetic, is infinite as there are no limits in applying S to successively generated numbers. Thus all the axioms together yield the concept of an infinite ordered set starting from 0.

This example makes it possible to explain the difference between creative and non-creative definitions. If we need a new object such as the function of addition, it must be introduced by a definition acting as an additional set of axioms, as shown above (Subsec. 3.2). On the other hand, if we wish to simplify some expressions without any change of the domain \mathbf{N} , we may introduce the term 'predecessor' defined as follows: x is said to be the predecessor of y if y is the successor of x . Nothing new enters the domain with this definition, only some statements may now be shortened or simplified (e.g., Axiom 3 can be shortened to the form '0 has no predecessor').

Thus the definition of the term 'predecessor' is asemantic in the sense suggested above (Subsec. 3.1), i.e., in the sense that its introduction does not change the domain of the theory. On the other hand, something is being changed in our possibilities of perception, now the same number can be perceived either 'from the front' (from the side of the successor) or 'from the back' (from the side of the predecessor). Thus the question arises, which may seem odd but proves vital for our issue, whether such a front and such a back of an object should be regarded as new objects. Obviously, they are no arithmetical objects but besides the domain of arithmetic there is also the realm which includes our ways of perceiving arithmetical objects. Should then the definition of the term 'the predecessor of a natural number' be regarded as creative in the sense of introducing new entities into the realm of arithmetical perception? The answer to be found is in the affirmative, but in order to give it a wording we should find a suitable expression to term the category of objects involved. This term is to be supplied by the following discussion.

4.3. The problem stated above emerges on numerous occasions, even at such an elementary level as definitions of propositional connectives. Though the formulas, e.g.,

$$p \rightarrow q, \quad \neg(p \wedge \neg q), \quad \text{and} \quad \neg p \vee q$$

refer to the same object in the propositionally interpreted Boolean algebra (the truth-function described in column 5 of T2 in Chapter Five, Subsec. 2.2), the way in which this object is perceived by a user of the truth-functional calculus is in each case different, so different that for some beginners a discussion is needed to convince them that of the equivalence of these formulas. But even an advanced student feels a difference, e.g., the second formula suggests how to deny a conditional, while the third suggests how to branch a conditional in a tableaux proof.

Undoubtedly, to each of these formulas there corresponds a different internal record in a connectivist model of mind, that is in such a model in which a record may be a function of a number of impulses coming from various centres (and not a static representation fixed in one place). Among such impulses are those corresponding to the structure of an expression as well as those corresponding to some associated ideas, e.g., the idea of denial, or the idea of branching. In other words, the object determined by a record involves some relational properties, and these are different in each of the cases discussed.

The problem of the nature, origin, and function of objects (often deserving to be called objects of consciousness) that correspond to internal records is too complex to be responsibly discussed on the occasion of another problem, as if on its margin; it requires serious treatment in a special study. On the other hand, it cannot be disregarded in the present discussion of definitions, for definitions require proof of the existence of an object, and then the question ‘what kind of object?’ cannot be avoided. Let the way out of this dilemma consist in a terminological suggestion which should make it easier to bring up the problem and to handle it in a possible further discussion.

I suggest the term **eidetic object** for an object which is somehow generated by an internal record. Such generation can be exemplified by our paradigmatic case of ostensive procedure (cf. Section

1 of this Chapter). As for this case, we know enough about the human nervous system to be sure that the ostended physical object has its counterpart in a very complex record written into nervous cells. It results from some processing of the retinal picture (fragmented into independent responses of a myriad of punctate elements, the rods and cones) which in its way to the visual centre passes through successive stages of abstraction. E.g., the brightness contrasts of the retinal image are in the visual cortex converted into contoured outlines by several neuronal stages of information processing. The final product is what happens to be called a **percept**, as in this statement by R. Jung: “The sensory raw material delivered by the receptors cannot become a percept without information processing over several levels in the brain. This includes feature extraction, spatial and temporal order and memory comparison which involves some reverberation and redundant resonance of sensory messages.”⁹

This percept, e.g., the picture of a red poppy flower being shown in a procedure of ostension to define ‘red’, has a set of properties which is by no means identical with the set of properties of the physical poppy, hence there are two different objects, viz., the poppy-percept and the physical poppy. What provides the content to be included the meaning of the term ‘red’ being defined is, obviously, the poppy-percept. For were the perceiver a daltonist, he would infer from the ostension that ‘red’ means grey (as belonging to his poppy-percept).

The concept of eidetic object is here meant as the generalization of the notion of percept such that it embraces also abstract entities. A poppy-percept is, then, an eidetic object, and a mental picture of 0 is an eidetic object, too, but the latter is no percept. It is assumed here that extralinguistic and extra-mental objects such as 0, belonging to an abstract domain, have some mental-biological counterparts, just as physical objects have their counterparts in percepts. Such counterparts, of either physical or abstract entities, are precisely what I propose to call eidetic objects. Some of them may consciously be realized by someone, and so become

⁹ See Jung [1973], p. 124. This quotation and some other data are taken from Popper and Eccles [1977].

what is called **objects of consciousness**, or following Brentano and Twardowski, *intentional objects*, but it does not mean any equivalence of these classes. Intentional objects form a proper part of the class of eidetic objects, otherwise we would not be able to account for the transformations of eidetic objects (exemplified above by the processing of visual information) since the greater part of the process, if not the whole, escapes realization by the subject in question. If at the final stage of such a processing, say stage n , an eidetic object emerges and is consciously realized (e.g., the idea of a wheel as conceived by a primitive ancestor of ours), it should be supposed that at the preconscious stage $n - 1$ there must have existed an eidetic object too, viz., that which has become the idea of a wheel after the last step of processing; otherwise, we should have believed in the *creatio ex nihilo*.

4.4. The concept of eidetic objects allows us a suitable location of what has been formerly called universals: they are meant as belonging to eidetic objects. Those universals which result from definitions constitute a network governed by the law of crucial importance for efficient arguing. I propose to call it the law of positive **conceptual feedback**.

The concept of feedback, taken from an engineering vocabulary, denotes the return to the input of a part of the output of a machine, system, or process; if such a return of the input increases or reinforces the quality of the former input, the feedback is called positive, otherwise (when bringing about a decrease) it is called negative. We shall have to do with a positive feedback in the realm of eidetic objects due to implicit definitions (the adjective will be omitted as only the positive feedback is to be taken into account). This phenomenon does not occur in the case of asemantic definitions, i.e., those being mere shorthands to introduce a new term, as an abbreviation for a longer phrase, without introducing a new object, and even without referring to any object at all.

The feedback characteristic of implicit definitions, which consists in successive increases of information conveyed by the terms involved, can again be exemplified by the axioms of Peano arithmetic. Suppose that the list of axioms (Subsec. 4.2 above) has been completed by the inductive definition of addition (Subsec.

3.2 above). Such definitions can be regarded as new axioms, thus functioning as an implicit definition.¹⁰ The meaning of the symbols ‘*S*’ (for the successor function) and ‘0’ influences the meaning of the addition symbol which comes later, after appending the definition of addition. However, the reverse is also the case: the latter symbol increases the content of the former ones, e.g., due to that definition the content of the term ‘0’ hints at the property that after adding 0 to any number this number does not change.

The inductive definition of multiplication which should come in the next step of building the system of arithmetic exemplifies a similar feedback.

$$\begin{array}{ll} y \cdot 0 = 0 & \text{the initial step} \\ y \cdot S(x) = (y \cdot x) + y & \text{the induction step} \end{array}$$

Again, the meaning of the previously existing symbols ‘0’, ‘*S*’, and ‘+’ is necessary to define the multiplication function (otherwise we would not be able to introduce multiplication), and this, in turn, enriches the meaning of, for instance, ‘0’ with the property of converting into 0 whatever is multiplied by it (that this is no trivial information can be seen in a classroom where some schoolboys have troubles in multiplying by 0 in spite of their knowing the facts referred to by the former axioms).

The set of implicit definitions identical with the set of axioms of a deductive theory furnishes an ideal model to be approximated in empirical theories and in everyday arguments. Empirical theories also have their axioms though, as a rule, they are hardly so completely listed and so systematically arranged as in mathematical theories. For instance, the statement that every planet revolves around a star belongs to unofficial, so to say, axioms of astronomy, and — as expected of axioms — it is part of an implicit definition of a planet; if someone denies it, he does not take the word ‘planet’ in its established sense.

The creation of empirical theories is an instinctive activity of every mind, not only the job of scholars. They are aimed at reducing the chaotic multitude of surrounding facts to few organizing

¹⁰ In fact, in some versions of Peano arithmetic the set of axioms includes the inductive definitions of addition and multiplication. See, e.g., the article ‘Arithmetic’ by W. Marek in *Logic* [1981].

principles from which facts could be deduced, and so explained or predicted. At each level of development, from magical beliefs of primitive tribes up to most sophisticated achievements of science, a theory consists of two interacting areas, that of definitions and that of factual statements, i.e., statements about facts, either experienced or guessed; the latter statements should be deduced from the former combined with definitions. The circumstance that definitions, being also axioms, are not systematized, are not even listed or consciously recognized, does not mean that they are not active; and the fact that deductions are often made unconsciously does not deprive them of the import of genuine deductions.

Let us focus on the definitional area of a theory. The conceptual feedback makes a system, or a network, out of the set of concepts recorded in one's mind. Let it be called **conceptual network**. Each conceptual record either directly or indirectly influences others and is influenced by them. Their functioning can best be described in opposition to that of definition which has been called *asemantic*; in the latter the definiendum is entirely passive, i.e., its whole meaning is given to it by the definiens, and the definiens, as far as its meaning is concerned, cannot take over anything from the definiendum. In an implicit definition the meanings of the term involved influence each other, as shown above in the example of arithmetical concepts in Peano axioms.

4.5. There is a variety standing halfway between normal and implicit definitions which is of special rhetorical consequence. Let it be termed **theoretical redefinition**. The meaning of the prefix *re*, as hinting at a *retro* action, is here related to the idea of conceptual feedback. The schema of the proceedings is as follows.

Phase 1. A new object which has a set of empirically observable properties, say E, is discovered and a new expression (not existing in the given language as yet) is proposed to name it; let it be the name 'A'.

Phase 2. As a result of continued research, a new set of properties, say T, not necessarily observable, is claimed to belong to A, on the grounds of some theoretical premisses, which proves so essential for the theory in question that the researchers decide to define A once more, this time in terms of T.

Phase 3. The decision-makers, however, do not have a full control of the communicative process, and the previous meaning of ‘A’ does not disappear from the linguistic usage. Now both sets of properties, that is E and T, are involved in the content of ‘A’ which amounts to the assertion that A is marked by both E and T. Were this assertion disconfirmed, it would be necessary to change the definition once more, but as long as it holds in the theory, it is cognitively fruitful as allowing us to deduce the occurrence of T on the basis of E, and vice versa. At the same time the meanings of terms referring to either set of properties enrich each other, like in a positive feedback — owing to the newly established, and definitionally sanctioned, relationship between E and T. This feature of mutual defining makes a theoretical redefinition similar to an implicit definition, while its origin resembles that of a normal definition.

Let the following story of defining the term ‘influenza’ illustrate that schema. First there was a prescientific conception which explains the etymology, namely the frightening epidemic dissemination of the disease was attributed to the *influence* of heavenly bodies, thus the definition of influenza was related to some astrological conceptions. With the decline of astrology no trace of this primitive meaning was left, and then the word was defined in terms of observable clinical characters listed as follows: “fever, prostration, severe aches and pains, and progressive inflammation of the respiratory mucous membrane.” Later, it was discovered that the disease is caused by a virus, and then biologists decided to redefine influenza as a disease caused by a virus with certain characteristics.¹¹ Thus there emerged a redefinition supported by a fertile and well-confirmed biological theory. The former definition, though, did not disappear (as once did the primitive astrological conception) but formed a whole with the new theoretical component, to the advantage of both the clinical diagnosis and the therapy based on theoretical etiology.

Such a picture of the activity of the mind complies with the **connectionist view** of the architecture of the brain (or mind) as a

¹¹ Part of this story is reported by Beveridge [1953], p. 86. The definition given above in inverted commas is found in Webster [1971].

system of “networks which consist of large number of multiply interconnected nodes. The nodes are simple, numerous and interact without supervision from a central processing unit. In the limit, all the nodes are connected; even if the limit is not reached, each node is connected with many others.”¹²

This is not to say that singular concepts correspond to singular nodes. We should rather suppose that a concept is recorded dynamically, i.e., as a state of perhaps a vast area of nodes whose firing up is a function of the set of incoming stimulations and the previous state of that node. The term ‘internal record’ so frequently used in this discussion is not bound to mean something like a print in a definite spot of the nervous tissue; it may mean any physiological state able to represent what we experience as a concept, e.g., a state of interaction among many nodes or centres.

For the present discussion it is not necessary to decide which of the rival models of the mind is true, but the connectionist model has a heuristic value for logico-rhetoric strategies. It helps us to remember the golden rule that what counts most in arguments is not forcing a single thesis or a single idea upon the audience but rather creating as many connections as possible between the point argued and the addressee’s conceptual network. This new point should be organically embedded into the conceptual network addressed, and that result would mean the full success of our argument. Therefore the argument should be preceded by a reconnaissance regarding that conceptual network — not, obviously, in its biological functioning (which will remain unattainable for some time yet) but in what we can recognize or guess on the basis of the verbal and non-verbal behaviour of our partner. The connectionist model as claiming a very dynamic picture of a conceptual network encourages us to try to modify it, provided there is a fitting previous recognition as to existing conceptual connections which, supposedly, correspond to some neuronal connections.

4.6. The accumulation of concepts devised throughout this discussion has reached that point at which we are able to express

¹² See Sterelny [1990], p. 169. The book contains the instructive chapter on connectionism, discussed as the alternative to representationism, the latter being also known under the names of classical cognitive psychology, classical artificial intelligence, the language of the thought model of cognition.

the main points of cognitive rhetoric. Let it be recalled, cognitive rhetoric, in contradistinction to emotive rhetoric, deals with the methods of how the arguer should induce his partner to the acceptance of a truth which is advantageous from the arguer's point of view; it is not necessarily a personal advantage of the arguer, it may be the good of a community, or a cause with which the arguer identifies himself, even the cause of truth itself. This is the very essence of any cognitive-rhetorical undertaking.

The preceding chapter paves the way for the present one through some limitative results concerning the import of reasoning for rhetorical enterprise. There it is claimed that human minds have so excellent an inferential machine in their brains that it in a way surpasses what can be offered by logical theories. Obviously, this machine is not infallible, but its complexity leaves little chance to theoretical logic to prove helpful in repairing defects by a direct intervention. Nevertheless, logical theories significantly contribute to studying and understanding the inferential activities of the mind; certainly they can achieve more than purely descriptive psychological theories, for a reasoning cannot be studied without the idea of inferential validity, and that is entirely due to logic. A better understanding of the mind due to the logical ideas and results should provide us with a basis for rhetorical reflection. Thus instead of direct precepts of how to efficiently argue, which in vain would be expected from theoretical logic, we obtain a vast assemblage of ideas to be further processed and developed from the rhetorical point of view.

The idea of logical validity belongs to those most important ones which are supplied by predicate logic. The other, of no less import from the rhetorical point of view, is the idea of generalization as examined in the preceding chapter by the method consisting in the formalization, even computer-assisted formalization, of some carefully chosen specimens of reasoning. This idea finds a direct application in the theory of definition as developed in this chapter. The analysis of reasonings shows the feature of instinctive generalization sanctioned in predicate logic by the generalization rule (also called the rule of introducing the universal quantifier).

After having gained such results owing to predicate logic, we need a bridge between its language and our ordinary languages, for

in the latter there is practically no trace of the use of bound variables, and there is, instead, the use of general names. Apart from the translation problem, successfully solved owing to some reflection on the relation between predicate logic and ordinary language (including the distinction of the strong and weak interpretation of universal statements), there is the semantic problem of what should correspond to general names in extra-linguistic reality, especially when such a name is being defined and therefore it should meet the requirement of existence. The course taken in this discussion consists in accepting the idea of general object, called also a universal, as the reference of a general name. A universal is what our mind strives for in any act of generalization, especially under the impulse produced by an internal record (as, e.g., the record of an innate idea of natural number brings about some arithmetical concepts expressed in suitable general names). Whether universals really do exist is a sophisticated philosophical problem which requires a careful treatment, and can by no means be solved as it were on the margin of another study. Therefore, instead of tackling this problem, I proposed a moderate course which involves the concept of **eidetic supposition** related to the concept of **eidetic object** (See this Chapter Subsec. 4.3). Thus we do not decide whether universals exist, but formulate instead some regulations concerning our ways of speaking about universals, so that people become able to understand each other and to argue in an orderly way.

Taking into account all those points, we can summarize the main lesson which cognitive rhetoric owes to this discussion as based on theoretical achievements of logic. It becomes clear that success in vindicating one's point in an argumentative dialogue depends on meeting two requirements to be subsumed under the title of the

Golden Rule of Rhetoric: *Show the thing itself, that is a universal, and give it the most illuminating name you can find in the language of your audience.*

The showing of the thing consists in a skilled combination of demonstrating suitably chosen specimens and an appeal to the supposed preconceptions of the addressee somehow present in his internal records. As for the latter, in some cases, like in teaching mathematics (according to the famous paradigm found in Plato's

Phaedo), one can rely on the supposed similarity of innate records existing in all human minds; in other cases one has to take into account cultural and individual varieties which should be previously recognized with an adequate degree of precision. As for the choice of specimens, there are several methods. In an ostensive definition it is a physical object to instantiate the universal referred to by the term being defined. In teaching geometry, an important role is played by drawings though they are by no means identical with the universals to which a proof refers. In other cases, e.g., when arguing that a deed is morally good, we should display some examples with the help of a narration which would best illustrate the intended concept of being morally good; such an exemplification may be supported by a theory in which the features attributed to the universal *good* could be shown with a proof.

Very interesting issues are connected with the postulate of choosing the most illuminating name in the audience's language.¹³ The adjective 'illuminating' may be not the best with regard to collocation rules of English, but I decided to suggest it because of some picture of mental or nervous activity to be supposed on the grounds of a model of the mind (e.g., one mentioned above in 4.5). According to that model, there are three distinct zones of the cognitive system (say, a human brain), one of them containing records of things, another one records of names, and the third being the intermediary between the two.¹⁴ What is specially important for our problem is the existence of the intermediary zone since this creates the possibility of interaction between the process of contemplating a thing and the process of looking for the proper name for the thing; it is this process in the mind of an addressee which can be controlled by the arguer addressing his audience. This part of our golden rule which encourages the arguer only to assist the illumination, which should mainly derive from the audience's own conceptual resources, is guided by that couplet fittingly quoted by

¹³ They would deserve a special chapter in this book, but objective requirements of theoretically treating a subject are not always reconcilable with practical circumstances of writing a book, therefore they are only briefly mentioned.

¹⁴ Such a model results from the research reported by Damasio and Damasio [1989], Damasio et al. [1990], and Damasio [1992].

Henry Newman in his *Grammar of Assent* [1870]: *A man convinced against his will is of the same opinion still.*¹⁵

The illuminative function of a name, as chosen to stand for the demonstrated thing, derives from its connections with other nodes in the conceptual network of the person in question. Suppose that the expressions *A* and *B*, similar (but not identical) in meaning, are considered as candidates for the name (in the addressee's language) of the thing *T* which has been shown to the addressee (with the help of a method from among those discussed above). There are reasons to maintain that the weight of connections of *A* and *B* with other expressions depends on the frequency of contextual co-occurrences as encountered by the person in question.

For example, when writing the last section of Chapter Four, I deliberated whether to use the adjective 'universal' or 'general', or else 'eidetic' to term the kind of supposition discussed in that section. The first and the second might have proved fitting because of my intention to attribute a universal, or (in other words) a general object to any name appearing in this supposition. However, these adjectives occur in logical contexts which, on account of some undesirable associations, would obscure the intended meaning. In the preceding section of the same chapter I discussed what I decided to call 'class supposition' to cope with the reference relation between a name and a class, and a class also possesses the feature of generality, while the universal class has the feature of universality, and neither feature was to attach to the supposition I wished to endow with an appropriate adjective. Also the associations with general names, general statements and universal statements, all having a considerable weight due to their frequent occurrences in logical contexts, must have acted as an obscuring factor. At the same time, the adjective 'eidetic' conveyed most desired contextual connections (at least for philologically skilled readers) because of

¹⁵ Again, as in a preceding footnote, it should be mentioned that only some accidental (in the sense of a theoretical import) circumstances prevented me from including a chapter which would take advantage of H. Newman's ideas about philosophical foundations of the art of argument. Newman's attitude is like that of Plato and Augustine, who emphasized the role of internal illumination which may be assisted by a teacher or arguer from the outside but finally the assent depends on mental processes of the addressed person himself.

its appearance in Plato's dialogues — in the very sense I intended (as explained in the last section of Chapter Four).¹⁶

Now, to take advantage of the above example, let *A* be read as 'universal', *B* as 'eidetic', while the thing *T* to be named is the kind of supposition which was the subject-matter of Subsections 3.4 and 3.5 of Chapter Four. The name 'eidetic' due to its associations, giving it the advantage in weight over the alternative naming, sheds light on the features of *T*, and that, in turn, reinforces the content issuing from the examples which show the relevant features of the thing in question (here, the discussed kind of supposition). Thus the realization of these features suggests an adequate naming, and the naming reinforces the vision earlier created by demonstration. This is what I suggest to call the illuminative function of the name. Supposedly, this is possible due to the brain zone acting like an intermediary between the concept zone, i.e., a network of records concerning things themselves, and the linguistic zone, i.e., the network of records of expressions (as graphical or acoustic forms).

4.7. Our Golden Rule, while summing up the multitude of concepts previously introduced and discussed, paves the way for the notion of rhetorical acumen, or rhetorical intelligence, as part of penetrative intellect.¹⁷ Now we can better see that the centre of gravity of intelligent arguing lies in the art of defining.

The cognitive situation in ordinary thinking is very different from that existing in logically arranged mathematics. In mathematics we strive for the ideal which consists in deducing the greatest possible number of theorems from the least possible number of

¹⁶ As remarked above, such a terminological decision should be substantiated by recognizing the conceptual network of the prospective audience. My recognition amounted to the guess (which may prove wrong) that those readers who are logicians are familiar with Beth's [1959] logical interpretation of the Platonic *eidōs*, while historians, philologists or philosophers are acquainted with Plato himself; German readers are supposed to be interested in Carl Friedrich von Weizsäcker's [1981] inspiring thoughts about the relation between the Platonic *eidōs* and modern science, esp. physics and biology.

¹⁷ These notions were introduced and discussed in Chapter One, Subsec. 2.4 in a preliminary way which finds its completion in the present comments based on the content of the Golden Rule.

axioms. This is why the chains of reasoning are so long and cumbersome for laymen who lack the necessary training. In everyday thinking concerned with empirical world we deal with an enormous multitude of general objects, i.e., universals, which are subjected to incessant processing and modifying in such an enormous mass that it is not possible for any thinking subject to control these processes in their entirety; only a very small part of them, like the tip of an iceberg, can be realized and consciously controlled by the subject in question, and due to that solution our brains are endowed with a colossal power of transforming information (even during sleep the brain continues to do a tremendous amount of work).

The point to be emphasized is that the logical correctness of those massive processes of transforming information is ensured through our perfect logical mechanisms of reasonings being fit enough to efficiently act also beyond our consciousness. Hence, if there are errors in our thinking (and, as we all know, there are many), the factor which is responsible for this deplorable state of affairs is not the inferential mechanism but rather some false axioms which amount to implicit definitions accumulated throughout the conscious and unconscious life of our mind.

We then reach the conclusion that the main task our intelligence should cope with is the task of properly forming implicit definitions. It is a real challenge to our human intelligence, and to artificial intelligence as well, should it match that of ours. The chance of meeting the challenge with success should increase owing to the ideas and findings to be expected from so many scholars active in this field. This essay is hoped to bring to its readers' attention a few ideas which are not very likely to be encountered by them in the writings of other authors.¹⁸

¹⁸ These comments ending the present chapter are meant not only as a résumé but also as a concise record of ideas to be developed in further research. In a sense, they end the whole essay since the next two chapters mainly illustrate the ideas discussed so far, doing that in a series of case studies. Those in Chapter Nine are to exemplify the points of Chapter Seven concerning reasoning, and those in Chapter Ten continue the present chapter also in the case study form.