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The Principle of Comprehension as a Present-Day Contribution to Mathesis Universalis*

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1. Problems and Conjectures

1. There is no doubt that the present day of logic starts with Gottlob Frege. This certainty is accompanied by the opinion, as expressed by J. Łukasiewicz (1935), J. van Hejienoort, I. Angelelli, Ch. Thiel (1982) and others, that with Frege mathematical logic (mathematical in its methods and its foundational task) suddenly came full-grown into our world.

In such statements the notion *suddenly* deserves attention. There is a maxim, believed by Leibniz, *Natura non facit saltus*. Believing or not in this most general principle, one may suppose that, at least, history does not make sudden leaps: *Historia non facit saltus*. The question whether it does or not, belongs to main issues in philosophy of history, especially history of ideas. But whatever is historian's philosophical view in this matter, as a historian he should try to bridge the gaps he perceives as if there were a continuous development, without any hiatuses. Only then, after he did his best looking for steadiness and filling gaps, his eventual statement of discontinuity, or even a revolutional change, can be accepted as reliable.

The problem of continuity in the development of logic is related to the question whether modern logic does belong to the same line of development in which Leibnizian logic is found, the letter being considered within the frame of *mathesis universalis*, that is, the 17th century program for unified knowledge.

Let me put these questions in a more figurative way. Let us imagine two configurations of points at a map of history of logic: (1) the program of universal mathematics in the 17th century; (2) the ideas characteristic of the first period of modern logic and foundational studies as conceived by Frege, Dedekind and Cantor. If there is a point in the latter which historically derives from a point belonging to the former, then there exists a line of development. Now, our task is *to trace such lines and to minimize gaps and jumps* that may occur in them.

2. The point chosen for the present study is the idea expressed by the principle of comprehension (of a naive set theory), viz.

PC $\quad (Ey)(x)(x \in y \equiv A(x))$,

* This essay is dedicated to Professor Jozef Iwanicki, my first teacher of logic, to whom I owe my commitment to the idea of mathesis universalis.

A being any well formed formula. Questions to be asked regard the relations of this principle to mathesis universalis. Does PC contribute to the realization of mathesis universalis as a program of unified knowledge? Does PC possess any anticipations within mathesis universalis?

As to the first of these questions, the answer proves affirmative because of the following facts: the program of mathesis universalis postulated integration of knowledge upon a mathematical foundation; set theory, based on the idea of comprehension, did succeed in integrating vast areas of knowledge, viz. mathematics and some related fields. Moreover, the philosophical import of the principle of comprehension has brought mathematics and philosophy much closer to each other, this being another integrative effect (cf. Beth, 1959, Küng, 1967, Fraenkel et al., 1973).

As to the second question, whether the principle of comprehension did appear in any form in the age of mathesis universalis, the issue is more controversial. There is a conjecture, stated by such a serious author as Frege himself, which reads as follows; there is a logical law which was ever used in Leibniz-type logic, the principle of comprehension being a consequence of this law (in this sense the principle may be said to be implicit in such a logic). The law in question, being among the axioms of Frege's "Grundgesetze" (axiom V) is as follows:

GG.V $\quad (x)(f(x) \equiv g(x)) \equiv \{x: f(x)\} = \{x: g(x)\}$.

Frege s historical comment regarding this law runs as follows.

> Wir können die Allgemeinheit einer Gleichheit immer in eine Werthverlaufsgleichheit umsetzen und umgekehrt. Diese Möglichkeit muss als ein logisches Gesetz angesehen werden, von dem übrigens schon immer, wenn auch stillschweigend, Gebrauch gemacht ist, wenn von Begriffsumfängen die Rede gewesen ist. Die ganze leibniz-boolesche rechnende Logik beruht darauf (Frege, 1893, sec. 9, p. 11)

Does in fact appear a counterpart of GG.V in Leibniz's logic? This is the question to be posed, according to the rule of searching for historical continuity. In what follows a Leibnizian principle that might be seen as a counterpart of GG.V is discussed. A factor disconfirming the Fregean conjecture is considered, namely Leibniz's treatment of abstract terms affected by his nominalistic tendency. This tendency, inherent in the Leibnizian version of universal mathematics must have been rejected to pave the the way for the further development of mathesis universalis. This has been done with the principle of comprehension, opening the way to higher and higher levels of abstraction.

Thus there appears a gap between Leibniz and Frege. However, it does not mean that the rule of historical continuity ought to be dismissed. Frege's achievements appear at an intersection of two lines of development: that of logic and that of mathematics. It may be the latter in which the continuity is to be traced; this conjecture is supported by the simultaneous appearance of the principle of comprehension with Frege, Dedekind and Cantor. A sketchy discussion of this fact is offered below, but a careful investigation deserved by this problem should be the subject of a new study.

2. The Leibnizian Version of Mathesis Universalis

1. The design of mathesis universalis, for short *MU*, was stated in the 17th century as part of the rationalistic philosophy of this time including a program of mathematization of sciences (see Weingartner, 1983). However, the significance of *MU* is not restricted to that period. It belongs to main ideas of Western civilization, its beginnings can be traced to Pythagoreans and Plato.

Immediate sources of the 17th century *MU* are found in the 15th century revival of Platonism whose leading figure was Marsilio Ficino (1433–1499), the author of "Theologica Platonica" He was accompanied by Nicholas of Cusa (1401–1464), Leonardo da Vinci (1452–1519), also by Nicolaus Copernicus (1473–1543). All of them may have taken as their motto the biblic verse, willingly quoted by St. Augustine, *Omnia in numero et pondere et mensura disposuisti*, Sap. 11, 21. The core of their doctrine was expressed in Ficino's statement that the perfect divine order of the universe gets mirrored in human mind due to mind's mathematical insights; thus mathematics proves capable of the role of an universal key to the knowledge; hence the denomination *mathesis universalis*

This line of thought was continued in the 16th century by Galileo Galilei (1564–1642) and Johannes Kepler (1571–163); it penetrated not only mechanics and astronomy but also medical sciences as represented by Teophrastus Paracelsus of Salzburg (1493–1541). No wonder that in the 17th century the community of scholars was ready to treat the idea of *MU* as something obvious, fairly a commonplace, before Descartes made use of this term in his "Regulae ad directionem ingenii". "Regulae" did not appear in print until 1701, hence the term itself could not have been taken from this source. In fact, it was used earlier by Erhard Weigel, a professor of mathematics in Jena (Leibniz's teacher) who wrote a series of books developing the program of universal mathematics: "Analysis Aristotelis ex Euclide restituta", 1658 (an interpretation of Aristotle's methodological theory in the light of Euclid's practice); "Idea Matheseos Universae", 1669; "Philosophia Mathematica: universae artis inveniendi prima stamina complectens", 1693 (see Arndt 1965).

2. The last of the listed titles involves one of the key concepts of the *MU* program: *ars inveniendi*, i.e. the art of discovering truths in a mathematical way. There were two approaches to this art, differing from each other by opposite evaluations of formal logic. According to Descartes, formal logic of Aristotle and schoolmen was useless for the discovery of truth; according to Leibniz, ars inveniendi was to possess the essential feature both of formal logic and of mathematical calculus, viz. the finding of truths *vi formae* (in virtue of form).

It happened that as influential a thinker as was Christian Wolff first endorsed Descartes' approach to the art of discovering (shared by E. Weigel), next that of Leibniz. His shift provides us with an instructive illustration of both approaches.

Wolff came to Jena in 1699 when Weigel's influence still lasted. Weigel denied syllogistic to have any power of finding truth and saw its use merely *in interpretatione textum, expositione et perspicua propositione veritatis iam inventae* ("Philosophia Mathematica", Praef. ad Lect.). At the same time Wolff found himself under the influence of E. W. von Tschirnhaus who, as Wolff commented in a later work, rejected syllogism as being *non tantum inutilis ad inveniendam, verum etiam ad examinandam veritatem* (Wolff, 1718, I, ch. 1, sec. 6). How did Wolff come to shifting his position?

In "Dissertatio algebraica de algorithmo infinitesimali differentiali", written and sent to Leibniz in 1704, Wolff stated: *Syllogismus non est medium inveniendi veritatem*. Leibniz responded; *Non ausim absolute dicere syllogismum non esse medium inveniendi veritatem* (letter of 21 Febr. 1705). This short and cautiously put comment changed Wolff's attitude to syllogistic, and may have induced him to read (or to rethink if he had read it earlier) Leibniz's (1684) paper in "Acta Eruditorum" where one could read the following views.

> Non contemnenda veritatis enuntiationum criteria sunt regulae communis Logicae, quibus et Geometrae utuntur, ut scilicet nihil admittatur pro certo, nisi accurata experientia, vel firma demonstratione probatum firma autem demonstratio est, quae praescriptam a Logica formam servat, non quasi semper ordinatis Scholarum more syllogismis opus sit (. . .) sed ita saltem ut argumentatio concludat vi formae, qualis argumentationis in forma conceptae exemplum, etiam calculum aliquem legitimum esse dixerim. (p. 81 in Erdm. ed.)

Wolff not only assimilated the Leibnizian appreciation of syllogism, but even went towards an extremity, disregarding Leibniz's provisos and regarding syllogism as the only means for mathematical reasoning: *demonstrationes geometricas ( ) si ad summam accurationem reducantur, constare ex syllogisms inter se connexis* (1718; II, ch. 2, sec. 26). He exemplified this claim e.g. in his textbook of logic (1713) when formalizing a geometrical demonstration by means of syllogisms alone and indicating how they help to find (not only to test) a correct solution (ch. 4, sec. 23 and 24).

Thus Wolff seems to have overlooked a crucial point in Leibniz's views on syllogistic: that syllogistic provides us with *a paradigm* which is as good as the algebraical paradigm of formal reasoning, but this does not mean that it exhausts all the forms of inference. However, the discussion of Wolff's views helps us to grasp the two essentially opposite tendencies among the followers of *MU*: that accepting formal logic as a crucial component of the mathesis, and that rejecting it altogether. It was the former which was to win and to be continued in the further development, and thereinafter we shall dwell upon it.

3. Thus, Leibniz's logical work lies in the point of intersection of the idea of universal mathematics and the theory of syllogism. Yet more lines of development cross with each other in the same point, viz. achievements of algebra, including notational innovations, and the projects of a universal language in which all human thoughts could be precisely expressed.

In the period preceding Leibniz's activities not only Descartes but many other authors as well contributed to the creation of algebra, e.g. G. Cardano (1501–1576), N. Tartaglia (1500–1557), F. Viete (1540–1603), S. Stevin (1548–1620). Even a new term appeared to designate this emerging branch of mathematics, viz. *analytica speciosa*, that is dealing with *species* of numbers, i.e. variables (instead of individual numbers as dealt with by arithmetic). Leibniz discovered the possibility of treating categorical propositions like algebraic equations whose one side was formed by juxtaposing subject and predicate (as if they were multiplied), the other side denoting either existence or non-existence (like in *propositiones secundi adjecti* of scholastic logic – see sec. III.4 below).

The idea of a logical calculus has derived from yet other sources, apart from *MU* and the syllogistical and the algebraic paradigm. There were still two intellectual movements, independent of each other, that contributed to Leibnizian logic. One of them tended toward the creation of a precise universal language called in many ways, e.g. *characteristica universalis, characteristica realis, lingua philosophica, characteristica rationis*. Leibniz (1666, item 89) considered some earlier projects of this kind, next he attentively observed and discussed those currently offered by some scholars from the Royal Society in London (esp. J. Wilkins and G. Dalgarno). This important story is told by Couturat (1901), hence there is no need to tell it once more.

Let us only note that the main logical principle on which such a language was to be built according to Leibniz, was not involved in those pre-Leibnizian designs of universal language. The principle in question demanded that all the notions of a language be divided into the simple primitives (*alphabetus cogitationum humanarum*) and the compound notions, the latter being analyzable and definable in terms of the former. This principle was developed within another scholarly movement in which Joachim Jungius (1587–1557) was involved as a prominent participant. To illustrate the extent of this movement, it may be of use to quote the following passage from G. Meier's letter to Leibniz (20 Jan. 1694).

> Placeat quod Christianum Thomasium regia incedere via arbitraris, idque congruit meae de terminis et de systematibus nostris positae sententiae. Afficior, quoties cogito Magnifici Blumii discursus, qui multos annos Joachimi Jungii (. . .) auditor fuerat. Is id unice per omnem vitam ursisse, id agitasse Jungium dicebat, ut ad primos terminos seu notiones primas juvenum et adolescentium studium referretur. Transire nos istas primas ideas, adeoque nunquam ad correctionem vel amplificationem veritatis rerum pervenire nec perventuros, quandiu non ad prima isthaec principia concedamus.
> (Akad. Ausg. 1979; series 1, vol. 10; p. 235)

Such views were in accord with the idea expressed by Leibniz in his academic exercise ("De principio individui", 1663) that *res sunt sicut numeri*. Here *res* meant notions, while statements about complex notions were to be derived from statements concerning their constituents by a combinatorial procedure analogous to multiplication of numbers.

Thus, by putting together that theory of complex notions and the idea of universal language, as well as the idea of universal mathematics, and else the syllogistical and algebraical patterns of formal reasoning, Leibniz became the first who succeeded in laying foundations under logical calculi. How such calculi are related to present-day logic, and whether they conform to Frege's conjecture as stated above (sec. 1.2), can be answered after we discuss the relevant Frege's points.

3 Did Leibniz Accept the Principle of Comprehension?

1. Owing to mathematical logic and set theory, mathematics has become more universal than it was ever before. It is no longer restricted to the study of quantity, as through centuries it was thought to be (especially in the Aristotelian school). Now it extends itself to what Leibniz called *qualitas* (e.g. in Couturat (ed.) 1903, p. 59), and nowadays we used to call it a structure (possibly meant by Descartes in "Regulae", rule IV, AT X, p. 59, when he defined *mathesis universalis* as dealing with *ordo et mensura*, i.e. structure and quantity). It is because of this extension that so many humanistic or social theories have their names prefixed by the adjective "mathematical", e.g. in economics, sociology, psychology, linguistics.

There are more resemblances. All the duties imposed upon the language of *mathesis universalis* are done by modern logic with set theory. Their language is symbolic what amounts to its being a *characteristica* in the 17th-century sense; it is capable of being employed by many sciences outside mathematics, hence it is *universalis*; its symbols correspond to concepts, not to sounds of a spoken language, hence it is conceptual, i.e. *realis* (note that often Leibniz called concepts *res*); it rests upon an ontological theory (being not unlike a Platonian doctrine), hence it also deserves the title *lingua philosophica* (as for these denominations, see sec. II.3 above).

Morever, to obtain *mathesis universalis*, such a language was to be completed by a calculus giving us formal rules of inference (*calculus ratiocinator*); and that is splendidly performed by our mathematical logic. If one desires to express purely philosophical discussions in such a language with an appropriate calculus, then he can resort to what is called philosophical logic, according to the 17th century program of mathematization of philosophy.

There was still another task for universal mathematics: before performing the universalizing and unifying role mathematics must have got united itself. Leibniz (1666, Intr., item 7) was of the opinion that hitherto existing mathematics was a heterogeneous aggregate of disciplines subsumed under the common denomination for the sake of convenience (possibly he meant the curriculum called *quadrivium* within the liberal arts). In the 17th century serious steps were made toward the integration of mathematics. Descartes' analytic geometry was the fusion of geometric, algebraic and functional thinking. Leibniz combined logic with algebra, tried to arithmetize logic, worked on geometrical interpretations of his algebraical calculi.

The present-day integration of mathematics, based on the set-theoretical foundations, was not exactly like that imagined by Descartes and Leibniz, nevertheless it was their program which has reached a realization. To render this fact, let me quote a suggestive statement of Fraenkel (1976).

> Set theory, as the most general field of mathematics and on account of its close connection with logic, has to fulfil the task of methodologically investigating and basing the primary concepts of mathematics in general, such as number, function, mapping, order, etc. and of hence deducing the fundamental branches of mathematics. As the most comprehensive contemporary exposition of mathematics puts it, *on sait aujourd'hui qu'il est possible . . . de faire deriver presque toute la mathematique actuelle d'une source unique, la Theorie des Ensembles.* It had been Cantor's explicit design to create by set theory "a genuine fusion between arithmetic and geometry"; set theory is fit for this purpose because its methods almost equally apply to contin-

uous and to discrete subjects, hence seem apt to span the gap between both domains. (p. 239; the French passage is taken from Bourbaki, 1954, p. 4).

Summing up, mathematical logic and set theory, possibly completed by philosophical logic, is universal as being capable of expressing all formal arguments; moreover, the basic notions of mathematics and some related disciplines are definable in terms of set-theoretical notions.

2. Provided that set theory with logic constitutes the present-day universal mathematics, the principle of comprehension (completed by the principle of extensionality), as being the intuitive basis of set theory, proves crucial for such mathematics. Moreover, the principle of comprehension specifies the nexus between propositional functions and sets, in this sense being a bridge between logic and set theory. In fact, it was Frege's purpose to come to a theory of sets and numbers by starting from the logical idea of a concept (a propositional function) and its extension. This transition is due to an equivalence which in "Grundgesetze" appears as the fifth axiom and from which the principle of comprehension is deduced; let it be rendered (in a current notation) as the two following conditionals:

Va $(x)(F(x) \equiv G(x)) \rightarrow \{x: F(x)\} \equiv \{x: G(x)\}$;

Vb $\{x: F(x)\} = \{x: G(x)\} \rightarrow (x)(F(x) \equiv G(x))$.

The statement Va is a version of the principle of extensionality and does not cause any difficulties. It is Vb from which the Fregean schema of comprehension

FC $G(x) \equiv x \in \{z: G(z)\}$

(with the help of a definition of membership) is deduced and which gives rise to the Russellian antinomy of classes. A more familiar form of the principle of comprehension (as in section I.2 above)

PC $(Ey)(x)(x \in y \equiv G(x))$

is obtained from FC provided that sets like $\{z : G(z)\}$ are genuine objects to which the existential generalization can be applied.

It is the conjunction of Va and Vb (in section I.2 above denoted as GG.V) upon which Frege commented that it was invariably employed, even if tacitly, whenever discourse was carried about the extensions of concepts as, according to Frege, it was in the Leibniz-Boole calculus. This comment suggests that the calculus in question rests upon the principle GG.V, and hence it involves something like FC or PC, that is a version of the comprehension principle. It is a suggestion that should be carefully examined as a serious historical conjecture.

3. If there is Leibniz's work in which a counterpart of GG.V is likely to be found, it is that entitled "Generales Inquisitiones de Analysi Notionum et Veritatum" written in 1686 (thereinafter abbreviated as "GI"). It constitutes a relatively complete and detailed exposition of a mature stage of Leibniz's logic; Leibniz himself was so much satisfied with it that he wrote at the margin of the first page of manuscript *hic egregie progessus sum*. Even its title indicates the problem in question, since GG.V, like the content of GI, is concerned with the relation between propositions (in the title referred to as *veritates*) and concepts (*notiones*).

Against this choice one might raise the objection that the logic of GI is intensional, while Frege's comment refers to extensional calculi. However, this objection can be dismissed, since Leibniz himself offers a key to the transforming of any of his intensional propositions into an extensional one, according to the following prescription.

Potest et alia consideratio institui, ut genus non ponatur esse pars speicei, ut paulo ante fecimus, quia generis notio est pars (vel saltem inclusum) notionis speciei; sed ut contra potius species sit pars generis, quia individua speciei sunt pars (vel saltem inclusum) individuorum generis. (GI, item 122)

Let it be added that Leibniz praised the scholastic improvement of syllogistics which consisted in the turning from the intensional to the extensional approach; this appraisal appears in his letters to Koch.

Vellem scire quis primus excogitaverit observationem de terminis distributis et non distributis. (2. Sept. 1708) . . . quae consideratio, jam nota quibusdam Scholasticis, insigne demonstrandorum modorum compendium praebet, et tamen ni fallor, apud Aristotelem haud extat. (31 Aug. 1710; quotations after Couturat 1901, p. 24)

The theory of term distribution, when combined with the theory of supposition, seems to have been the first step towards the concept of extension, as opposed to that of intension: a *terminus distributus* meant a term taken for (*supponens pro*) all the things falling under it; note that this distinction was not stated until the appearance of Port Royal Logic, 1662.

4. Before a counterpart of Frege's GG.V is found in GI and duly considered, one should explain in what sense the extensionally interpreted Leibnizian logic can be regarded as a calculus (*die rechnende Logik*, as Frege put it), viz. a calculus of classes.

Leibniz was the first who discovered the possibility of transforming categorical propositions into formulas being either equalities or inequalities like algebraic expressions. He treated the terms of a categorical proposition as the arguments of an operation which was expressed by their juxtaposition and defined by the laws $AB = BA$ (like a law for multiplication) and $AA = A$ (unlike multiplication; see Couturat 1901, p. 321). Such a rendering of categorical propositions required a preparatory step, and that was suggested by a scholastic theory.

The juxtaposed terms form one side of an equality (inequality) while the whole expression, owing to a structural similarity, can be obtained from a *propositio secundi adjecti* as considered in scholastic logic. The core of Leibniz's discovery consists in noticing an analogy between such a structure and the structure of algebraic expressions. According to the theory in question a *propositio secundi adjecti* (that is, composed of two constituents) can be always obtained from a *propositio tertii adjecti* (composed of three constituents, i.e. two terms and the copula) according to the rule which can be illustrated as follows.

p. tertii adjecti *Circulus* [i.e. figura plana ad unum aliquod punctum eodem modo se habens] *est figura plana*.
p. secundi adjecti: *Figura plana ad unum aliquod punctum eodem modo se habens est* [i.e. existit]. (GI, item 144)

If the existential *est* ist rendered by the Boolean symbol "$\neq \emptyset$", and *non est* by "$= \emptyset$", then it will be seen how this calculus of Leibniz is related to the modern theory of classes. When applying such a translation, one finds that the following reduction of *tertii adjecti* to *secundi adjecti*, as proposed by Leibniz, can be read in terms the modern class-theoretical interpretation of the four categorical propositions.

> Habemus ergo propositions tertii adjecti sic reductas ad propositiones secundi adjecti:
> Quoddam A est B, dat: *AB est res*.
> Quoddam A non est B dat: *A non-B est res*.
> Omne A est B dat: *A non-B non est res*.
> Nullum A est B dat: *AB non est res*. (GI, item 151)

Obviously, *est res* means the same as *est ens*, i.e. *est (existit)*.

5. The first step in searching for a counterpart of the Fregean GG.V consists in the observation that a universal and affirmative categorical proposition can be rendered as a hypothetical proposition, for instance

Omnis circulus est uniformis can be translated into
Si A est circulus, sequitur quod A est uniformis. (GI, item 143).

How important this mode of transformation seemed to Leibniz, can be seen in the following passage.

> Si, ut spero, possim concipere omnes propositiones instar terminorum et omnes Hypotheticas instar Categoricarum, et universaliter tractare omnes, miram ea res in mea characteristica, et analysi notionum, promittit facilitatem, eritque inventum maximi momenti. (GI, item 75)

In the same treatise Leibniz stated a principle which tells us how to reduce hypothetical propositions (being equivalent, as stated above, with some categorical propositions) to certain terms. This principle reads as follows.

> Propositionem ex propositione sequi nihil aliud est quam consequens in antecedenti [read "antecedens in consequenti", according to the extensional interpretation] contineri ut terminus in termino, atque hoc modo reducimus consequentias ad propositiones, et propositiones ad terminos. (GI, item 198, 8°)

According to what has been said above, a *consequentia*, that is a conditional of the form *Si A est F, (sequitur quod) A est G*, is equivalent with a categorical proposition of the form *Omnis F est G*, and this, in turn, can be replaced by a proposition (*secundi adjecti*) of the form *F non-G non est ens*. Morever, when the terms corresponding to the antecedent and the consequent of the conditional in question are formed by means of the abstraction operator, one obtains the following versions of the statement quoted above (from item 198) which will be referred to as the principle of propositions-to-terms reduction, for short PTR.

PTR** $(x)(F(x) \rightarrow G(x)) \rightarrow \{x: F(x)\} \cap - \{x: G(x)\} = \emptyset$;

that is (after resorting to the idea of inclusion, found also in Leibniz, e.g. in item 122 quoted above in sec. III.3):

PTR* $(x)(F(x) \rightarrow G(x)) \rightarrow \{x: F(x)\} \subset \{x: G(x)\}$.

The statement PTR* allows to obtain a new one, with the equivalence in the antecedent:

PTR $(x)(F(x) \equiv G(x)) \rightarrow \{x: F(x)\} = \{x: G(x)\}$.

Owing to the above reconstruction, one finds in Leibniz a statement identical with Frege's axiom Va, that is the principle of extensionality. In fact, Leibniz required this principle for his algebraic treatment of categorical propositions. It is possible that Frege's conjecture was by him stated with regard to the whole principle GG.V without distinguishing its component parts Va and Vb; in fact, the conjecture is true for Va. Is it true for Vb too? It is our next question to be answered.

6. When reading GI and other Leibniz's writings, one finds nothing that would resemble either VB or the principle of comprehension. Though this negative result cannot be a final disconfirmation, there is another argument against the conjecture that Leibniz accepts a statement similar to the principle of comprehension. The argument derives from his philosophy of language and its ontological assumptions.

Leibniz had philosophical reasons to claim that abstract terms should be eliminated from his *characteristica universalis*. When speaking of abstract terms, he meant the names of properties, not of sets, while set theory is concerned only with the latter. However, what is essential in his argument can be also applied to sets.

The argument in question is to effect that abstract terms, once introduced, would produce an infinite series of supposed abstract entities of higher and higher levels. This consequence seemed so paradoxical that it was used to refute its premise, viz. the acceptance of abstract entities. Here are Leibniz's own words.

> Placet removere hic conceptus Abstractos tanquam non necessarios, praesertim cum dentur abstractiones abstractionum. Et pro calore considerabimus calidum, quia rursus posset aliqua fingi *caloreitas*, et ita in infinitum. (Couturat 1903, p. 512; from the text entitled "Introductio ad Encyclopediam arcanam sive initia et specimina Scientiae Generalis")

Thus Leibniz proves to be a nominalist. His nominalism consists in the claim that there is only one ontological category of entities and, correspondingly, only one semantic category of names. It is worth noting that in the Leibnizian metaphysics individuals are found at the same ontological level as ideas are, each individual being an infinite collection of ideas; on the other hand, universals are either finite collections or no collections at all; the latter is true of simple primitive ideas. This may explain why Leibniz could not go beyond his one-levelled theory of classes (or ideas) toward a more-levelled predicate logic.

Such a nominalism must have been fundamentally opposed to something like the principle of comprehension, as this principle opens the way to the infinity of levels of abstract entities. Hence one can conclude that the principle of comprehension does not appear in that line of development which leads from Leibniz to Frege, even if other points are shared by these thinkers.

This conclusion may be objected on the grounds that there exists Leibniz's text which sounds like the principle of comprehension, viz. a premiss in "Demonstratio existentiae Dei" as contained in "De arte combinatoria". This statement, called *postulatum*, reads as follows.

> Liceat qoutcunque res simul sumere et tanquam unum totum supponere. Conceptus partium est, ut sint entia plura, de quibus omnibus si quid intelligi potest (...) excogitetur unum nomen, quod appellatur *Totum*. Cunque datis quotcunque rebus, etiam infinitis, intelligi possit, quod be omnibus verum est (...), licebit unum nomen in rationes ponere loco omnium: quod ipse erit *Totum*

When taken literally, it sounds like Cantor's definition of a set as resulting from a multitude due to its comprehension by a thought, grasping this multitude as one whole. However, Leibniz's above statement, when considered in its context, proves to be concerned with a very special case, when a multitude of things sharing a property (that all of them are being moved) at the same time constitutes one physical body (the aggregate of bodies that move and are moved). Hence the whole in question is rather a whole in the sense of Leśniewski's mereology than in the Fregean or Cantorian sense.

4 Concluding Remarks

If one takes into account some features of mathesis universalis as designed in the 17th century and finds certain similar features in modern logic with set theory, then one can introduce the notion of Mathesis Universalis (let it be distinguished by the capital initials) as an abstractum obtained from two historical concreta: that of the 17th century and that of our century; such a procedure was applied by Scholz (1934; cf. Scholz, 1961). The present essay deals with problems of thus conceived Mathesis Universalis. The main problem of the essay, as expressed in the title, splits into the following questions: (i) does the comprehension principle constitute a modern component of this Mathesis; if it does, then: (ii) is it quite a new contribution, or rather the continuation of a thought originated within the frame of the 17th century mathesis universalis?

The point (i) can be better understood in the context of Pascal's distinction of *esprit de geometrie* and *esprit de finesse*. This distinction gives rise to the question whether *esprit de finesse* can be helped by mathematical thinking; if so, there are greater chances for universal mathematics than there were in the case of answering in the negative. Inasmuch as a mathematical principle has a philosophical significance, it appears that philosophical discussions can be helped by *esprit de geometrie*. In fact, the comprehension principle has an indirect philosophical significance which is the following: the theory of sets has wide applications in philosophy and social or behaviorial disciplines (esp. linguistics), hence the principle of comprehension has a share in these applications. There is a direct bearing as well: the principle of comprehension shows that mathematics must presuppose notions which comply neither with nominalistic nor empiristic program of science, but rather with a program being like Platonism. Hence the question (i) is answered in the affirmative: the principle of comprehension belongs to a body of notions and statements which is mathematical as to its method and content, while its implications are both mathematical and philosophical, the latter bringing about a mark of universality.

The question (ii) was discussed in the context of Frege's conjecture that the axiom V of "Grundgesetze", referred to as GG.V, was tacitly assumed in the Leibnizian calculus. When GG.V, being an equivalence, is split into two implications Va and Vb, it appears that something like Va, namely a version of the extensionality principle, occurs in Leibniz's logical writings, while Vb, from which the principle of comprehension is derivable, is completely lacking in them; moreover, the principle of comprehension would oppose Leibniz's philosophical assumptions concerning the abstract entities. Hence, the principle in question, seen as a contribution to Mathesis Universalis, proves to be a present-day contribution without any anticipations in the 17th century.

Among the other conclusions of this essay is the statement that Leibniz's mathesis universalis derives both from the Platonian tradition and from the scholastic logic and methodology (see Iwanicki, 1933), especially the theory of *distributio terminorum* (as preparing the extensional approach) and the theory of *propositiones secundi adjecti* (as preparing operations on terms); this double influence is worth noting, as some authors disregard the former source, while the others disregard the latter.

The answer to the question (ii) may seem disconfirming the law of historical continuity as stated at the start of this essay. Let me note, however, that in this study only one line of development, viz. that of logic of classes was taken into account; since set theory was mainly developed within mathematics, it may be this line of development in which a continuity will appear. The principle of comprehension was at the same time stated by Cantor (1883), Dedekind (1888) and Frege (1893), independently of each other; such a simultaneous emergence of results is usually preceded by a steady collective effort of thought (as for the development of a collective intuition, see Wilder, 1981, ch. 7, laws 1, 2, 7, 9, 14, 20).

It was the internal need of the 19th century mathematics that caused the bold flight into abstraction, not dreamt by the 17th century philosophers, and that led to the Cantorian heaven of the infinity of abstract beings. Thus a new period of Mathesis Universalis was initiated in which both mathematics and philosophy *egregie progressae sunt*.

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