Witold Marciszewski

POLISH NOTATION, WELL-ORDERING, AND PRAXEOLOGY

Introduction

When looking for a formal model of human action, apt to be rendered in an algorithmic way, one may find a promising possibility in a generalization of syntactic principles underlying the Polish notation. The present paper drafts such a study.[1] That syntactic theory can shed light on more general problems of intelligent behaviour as investigated in cognitive science. Moreover, logicians have their own motivation to trace connexions between syntax and general laws of thought and action.

To wit, Cantor's well-ordering theorem, to the effect that **it is always possible to arrange every well-defined set in the form of well-ordered set**, was regarded by him as a *fundamental law of thought, rich in consequences and particularily remarkable for its general validity* – Cantor [1883] (p. 169, commented by Hallett [1984], p. 73, 155 ff, etc.). The same view was expressed by Zermelo and, in a sense, by Hilbert. There are some questions as to interpretation of this principle, e.g., what the phrase "well-defined" should mean, but there is a fairly common agreement as to its validity in the realm of mathematics.

What about its validity in the empirical world? In any finite domain of physical entitieies they can be well-ordered in a spatial sequence. It seems also reasonable to look for other empirical domains in which the well-ordering principle could be applied. Among those worth considering there are collections of well-formed strings of a language.

Obviously, Cantor's principle holds for such collections, in particular that defined by the grammar governing Polih notation; its principles are developed in what nawadays is known as a version of categorial grammar, going back to Kazimierz Ajdukiewicz [1935] (cp. comments by Buszkowski [1996]). As far as other grammars are concerned, the problem deserves a separate research. What is examined in this study, it is a praxeological generalization of that application of the ordering principle which holds for Łukasiewicz notation, as introduced by him in *Elements of Mathematical Logic* [1929].

1. A programme for syntax of action

Polish notation has found fruitful applications in computer science, especially its versions termed as reverse Polish notation (used in machine codes and such programs as PosScript), and Cambridge Polish notation (used in programming languages of higher order as LISP); the latter

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reintroduces perentheses without losing, though, the syntactic principle characteristic of this notation. When addressed to computers, a string of symbols in Polish notation does not express any truth about the world. Instead, it forms a sequence of imperatives to control machine's work.

These applications are motivated by greater simplicity (from computer's point of view) of parentheses-free strings in machine codes, and are due to the main syntactic idea underlying the notation. It is the idea of a hierachical sequence of operations, i.e. functions in the mathematical sense, being materialized in machine procedures. This hierarchy is what makes it possible to discern syntactic constituents without segmenting with parentheses, provided that the number and order of arguments of each functor (i.e. function sign) is previously defined in the vocabulary of the language in question. If the number of arguments is not given as, e.g., in LISP, then the language in question combines the use of parentheses with the syntactic order of Polish notation (Cp. Robinson [1979]).

The behaviour which is controlled by instructions recorded in a machine code provides us with a fitting model to be applied in cognitive science and biology. Human behaviour can be considered as realizing a program, in particular a program for planning a future action.

There is a widely known case of Koehler's chimpanzee Sultan who fitted a bamboo stick into another, after some futile attempts to solve the problem of grasping fruit that was out of his reach. In spite of the whole distance between a human and an ape, a human would react in a similar way, as in the case in question this is the only correct solution; therefore when reconstructing the structure of that action we are entitled to make use of our human thought-experiment. (Cp. Marciszewski [1994], p. 143 ff).

The planning proceeds from more general to more concrete statements concerning what should be done; the latter express ideas which cannot arise unless the problem is stated in general terms. In the following list of planning steps, the longer the sequence of digits, the more concrete step is numbered by it.

1. to have a tool adjusted to the distance from the agent to the fruit
1.1. to shorten the distance
1.2. to lengthen the stick (serving as the tool)
1.1.1. to come closer to the fruit
1.2.1. to join two sticks
1.2.1.1. to fasten stick no.1 to stick no.2
1.2.1.2. to fasten stick no.2 to stick no.1.

All the numbered items refer to actions being means which produce the desired result. The result is an action of higher order, e.g. the shortening of a distance which results from coming closer (by making some steps) to the object in question; the latter (1.1.1) is a concrete means to realize the former (1.1) as specified in a more general way.

Actions are arguments of the following functions:

*⇒ assigns the desired result to planned means;

n & combines two or more actions into a whole;

::: is the function which can be called sequential conjunction of more than two arguments; for instance, :::xyz means that x, y and z form a compound process, in which y follows x, and z follows y.

Thus the course of thought in Sultan's head could take the form described by the following formula (in which, for short, dots between digits are omitted).

::: [*⇒(&1211,1212)121] [&(*⇒121,12)(*⇒111,11)] [*⇒(&11,12)1]

This is to mean the following. The combination of actions 1211 and 1212 which results in 121 (that is, joining two sticks) is followed by the situation in which the fact that both old sticks are joined results in the making of the new stick, which is suitably longer. This, in turn, is followed by the situation in which both previous results make the agent equipped with the tool appropriate to the task in question; hence, again, the function denoted by the asterisked arrow assigns the desired result (to have a tool adjusted to the task) to the means being successively obtained, due to the actions having been undertaken.

Let it be noted that parentheses and spaces have been here added for the following two reasons: (i) for a greater perspicuity, (ii) as a provisional device which is required only then when the language in question is not sufficiently defined (e.g., as being created *ad hoc*, just to provide an example). In such a case, we need parentheses in order to indicate how many arguments should occur within the scope of the given functor, since this is not defined in an available vocabulary, as is defined, for instance, in the language of propositional calculus, arithmetic, etc. Obviously, in such a case the term ‘Polish notation’ ceases to be equivalent with the term ’parentheses-free notation’, but the original purity of Łukasiewicz’s idea can be restored, provided that kind of vocabulary in which the number of arguments following each functor is defined, and firmly fixed.

This is just an example of the syntax of planning actions. It represents only several functions, whereas there may be much more of them. A systematic study of such grammars should lead to a theory of action, or praxeology (in the sense given this term by Ludwig von Mises [1949] and Tadeusz Kotarbiński [1948], [1995]). The constraints imposed by syntactical principles of Polish notation should grant planning at a high level of preciseness because of the necessity of defining all the functions involved, and putting each of them into an exactly defined place in the string.

2. “That” – a test for Polish notation and a device in planning

When recommending Polish notation for extra-linguistic applications, the author is bound to test it in a linguistic area more challenging in this respect than are languages of logic and mathematics. There is in natural languages a syntactic construction which appears as headache if one tries to handle it with the Polish notation grammar, namely reported speech.

Let the sign “†” represent the particle “that" as involved in the string nv†s, where “n" represents a name, “v" a verb from the class containing “says", “believes", “knows", etc., and “s" hints at a sentence. There is a combinatory method to obtain various syntactic structures, and among them the following (quotation marks omitted for perspicuity).

[1] †(vn)s ... † links the sentences vn and s, functioning like a connective.
[2] (†v)ns ... † makes a new functor out of v, and this functor, in turn, combines n and s into a sentence.
[3] vn(†s) .... makes a name out of s, while v acts as functor to combine the names n and †s into a sentence.

Each of these interpretations is defended by some authors (as discussed by Marciszewski [1988]). In fact, each can be accepted as rendering one of possible intentions of the reported speech. Let us discuss the following statement in the Bible (Gen. II, 18).

The Lord God said, It is not good that the man should be alone.

Obviously, there is implicit † after “said". This may mean that the author reporting God’s saying treats “said" as abbreviation of “says that", hence “that" is a part of “says that"; there are reasons to interpret “says" as having category s/n, while † transforms it into category s/ns. This exemplifies form [2].

Under another possible interpretation, † fits into category n/s since the phrase “that the man should be alone" is the name of the situation of which it is predicated “is not good"; this is represented by schema [3]. Parsing [1] may be less convincing for a logician, nevertheless it is treated by linguists as one which renders the function of “that" as a connective to combine two sentences.

Interpretation [3] hints at the reported speech as a device in planning which is characteristic of human intelligence. In planning a future course of action, a human being creates a scenario, a story which he or she tells himself/herself. Such a story may start form the phrase “Let us imagine that”. Here “that" means something like “the following", and stands for the process being imagined. Thus it can be treated as making the name of such a process, in accordance with parsing [3].

Interestingly enough, it is that syntactic function of “that" which the researcher of intelligence William H. Calvin [1994] distinguishes as being specific for human cleverness. He means the capability of embedding a sentence into another sentence, like that nesting done in a rhyme for children, namely:

This is the farmer sowing the corn
That kept the cock that crowed in the morn.

Here “that" forms the name of an entity described by the sentence being nested. In a similar way we name imagined future events in the course of planning. This ability of nesting, according to Calvin, is what clearly distinguishes human language from what one may call animal languages. More on this subject is said in the next Section which should sum up the present discussion.

3. Syntax as a constitutive factor of human intelligence

It was stated above that the grammar underlying Polish notation can improve planning of actions, and so make them more likely to succeed. Since successful (in a long run) activity is a proof of intelligence, the above statement accords with the tenets, appearing in cognitive science, that intelligence derives from human syntactic abilities. This point is expressed, e.g., in the following statement found in Calvin [1994].

> Language is the most defining feature of human intelligence: without syntax — the orderly arrangement of verbal ideas — we would be little more clever than a chimpanzee. Something very close to verbal syntax also seems to contribute to another outstanding feature of human intelligence, the ability to plan ahead.

Calvin does not define the relation of "closeness" between verbal syntax and the ability to plan ahead. Neither goes he deeper into the very notion of syntax. The present essay hints at a method of substantiating such a theory through a more elaborated statement of grammar and checking it both against specimens of planning actions and against more involved structures of natural language. This is specially worth trying in the face of so far-reaching Calvin’s claims as the following.

> To understand why humans are so intelligent, we need to understand how our ancestors remodeled the ape symbolic repertoire and enhanced it by inventing syntax. [...] Human planning abilities may stem from our talent for building syntactical, string-based conceptual structures

> larger than sentences. [...] In this way, syntax raises intelligence to a new level. By borrowing the mental structures for syntax to judge other combinations of possible actions, we can extend our planning abilities and our intelligence.

However, as in the old riddle whether the egg or the hen was earlier, the reverse may be true, to wit that it was human intelligence that has brought syntactically organized language. There are good reasons to share Calvin's view as to the similarity of syntactic operations to planning and other intelligent actions. As to the priority of syntax, from which other performances of intelligence would derive, the question cannot be answered until the programme for the syntax of actions, as sketched above, becomes a full-fledged research project.

Anyway, if there is any grammar at all whose principles approximate those of planning structures in intelligent actions, it should take advantage of the concepts of operation and its operands, or functor and its arguments. Such constituents form an order which reflects the order constituted by the relation of means to ends; namely, the operand corresponds to a means while the whole formed out of it by the operator is what corresponds to an intended result.

The merit of Polish notation consists in showing that such a partial order can be transformed into a linear order, thus making our active lives more similar to the Cantorian ideal of well-ordering.

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